This article presents the design of a robust force controller for a hydraulic actuator interacting with an uncertain environment via quantitative feedback theory (QFT). After the derivation of a realistic nonlinear differential equation model, a linearized plant transfer function is developed. The effects of nonlinearities are accounted for by describing the linearized model parameters as structured uncertainty. The impact of environmental variability as well as variations in hydraulic component parameters are also included as uncertainty in the model. The QFT design procedure is carried out to design a robust controller that satisfies performance specifications for tracking and disturbance rejection. The designed controller enjoys the simplicity of fixed-gain controllers, is easy to implement, and at the same time is robust to the variation of hydraulic functions as well as environmental stiffness. The controller is implemented on an industrial hydraulic actuator equipped with a low-cost proportional valve. The experimental results show that robust stability against system uncertainties and under varying conditions is achieved and the performance goals are satisfied.

Introduction
Many industrial applications, such as manufacturing automation and material handling, involve interaction with the environment. In these applications, force control is required. Hydraulic actuators are advantageous for such ap-
plications because of their high force-to-weight ratio and fast response time. Additionally, they are able to maintain their loading capacity indefinitely, which would usually cause excessive heat generation in electrical components [1]. Unlike in electric actuators, however, force control in hydraulic actuators is a difficult problem [2], [3].

In a hydraulic actuator, the control signal activates the spool valve that controls the flow of hydraulic fluid into and out of the actuator. This flow in turn causes a pressure differential buildup that is proportional to the actuator force. Even if the spool valve dynamics are ignored, the control signal fundamentally controls the derivative of the actuator force and not the force itself. Furthermore, hydraulic systems are highly nonlinear and subject to parameter uncertainty; parameters change with time as a result of variations in operating conditions and component degradation. For example, the supply pressure is subject to variation that may be generated by the operation of other actuators in a multiuser environment [4]. The flow and pressure coefficients, characterizing fluid flow into and out of the valve, are functions of load and supply pressure and can vary under different operating conditions [5]. Also, the effective bulk modulus in hydraulic systems can significantly change under various load conditions, oil temperature, and air content in the oil [6]. Design of a controller in the face of such a range of parameter variations and disturbances is challenging. This article presents the application of QFT to the design of a robust force controller for hydraulic actuators.

In the literature, several force control strategies have been proposed for hydraulic actuators. Conrad and Jensen [2] used combinations of velocity feedforward, output feedback, and a Luenberger observer with state estimate feedback for force control of a double-rod hydraulic actuator. The simulation and experimental results for a constant setpoint force showed superior performance of the proposed method over conventional (P or PI) force feedback controllers. However, the variations of load and supply pressure were not considered in their study.

Chen et al. [7] designed a sliding-mode controller for a single-rod hydraulic actuator interacting with a spring as an environment. Using position, velocity, acceleration, force, and pressure feedback, the variable-structure controller proved to be capable in both static and dynamic force control tasks. The effect of servo-amplifier gain variation was also examined; however, the effect of variations in environmental stiffness was not studied.

Sun et al. [8] employed a sliding-mode controller with a perturbation observer for a single-rod electrohydraulic system. The effect of cylinder position and velocity on the pressure dynamics was considered as perturbation to the control model, which was estimated by an observer. The experimental results verified improved steady-state and transient performance as compared with traditional proportional-integral-differential (PID) controllers.

Adaptive control strategies have also been considered for hydraulic force control. Liu and Alleyne [9] developed a switching control scheme using a Lyapunov-based adaptive law to reduce parametric uncertainty. The implementation of the controller, which is based on the measurements of position, velocity, acceleration, pressure, and spool displacement, showed good performance for high-frequency force/pressure tracking.

Wu et al. [10] applied a generalized predictive control algorithm to a hydraulic force control system. The controller was experimentally evaluated for various environmental stiffnesses and setpoints. The method, however, relies heavily on online parameter estimation and consequently is computationally expensive.

Laval et al. [11] used an $H_\infty$ approach to robustly control the force exerted by a double-acting symmetric hydraulic cylinder with a servovalve. The importance of uncertainties in the environment, measurement, and nonlinearities on the performance of hydraulic force control systems was highlighted. Limited test results, demonstrating the achievement of a stability/performance trade-off utilizing an $H_\infty$ approach, were presented.

Despite the existence of a great number of force control concepts, methods, and algorithms, there is still a large gap between theory and industrial practice. The reasons have been ascribed to the poor industrial control architecture, which does not allow the implementation of sophisticated algorithms [12]. In this article, we employ the QFT technique to design an explicit force controller for an industrial hydraulic actuator. The goal is to arrive at a fixed-gain controller that: 1) is of low order and easy to implement, 2) is robust against uncertainties in both environmental stiffness and actuator functions, and 3) does not require exact knowledge of the system’s parameters.

QFT is a robust controller design methodology aimed at plants with parametric and unstructured uncertainties. The concept was first introduced by Horowitz in the early sixties and was later refined by him and others into a technique [13]-[15]. QFT emphasizes the fact that feedback is only necessary because of uncertainty and that the amount of feedback should therefore be directly related to the extent of plant uncertainty and unknown external disturbances. Minimizing the cost of feedback, as measured by the amount of controller bandwidth, is the main objective of QFT [16]. Therefore, the plant uncertainty and the closed-loop tolerances are formulated quantitatively so that the cost of feedback can be assessed at each stage of the design process [17]. (See the sidebar “An Overview of QFT” on p. 68.) The method has been applied to a wide range of engineering problems, including flight control [18], robot position control [19], and manufacturing systems [20]. Regarding the application of QFT to hydraulic systems, the technique was used to control an electrohydraulic actuator as part of a flight control system by Pachter et al. [21]. The controller’s efficacy was validated by simulation results. Thompson and Kremer [22] developed a QFT controller for a variable-displacement pump system based on a linearized model with...
An Overview of QFT

Quantitative feedback theory (QFT) is a frequency domain method intended for practical control system design given robust performance specifications. Its goal is to design a low-bandwidth controller that satisfies performance specifications, despite system uncertainties and disturbances. A low-bandwidth controller is a key issue in any practical design to avoid problems with noise amplification, resonance, and unmodeled dynamics. A brief outline of the classical QFT problem is presented here. A full treatment is given in [13] and [15]. Short overviews can also be found in [28] and [29].

QFT Configuration

A two degree-of-freedom system is typically assumed for the QFT technique when only output Y and command input R can be measured independently. Plant \( P \) contains system uncertainties and is exposed to unknown disturbance \( D \). \( G \) is the controller, which can help reduce the variation of the plant output due to uncertainties and disturbances, while \( F \) is used merely as a filter to tailor the response to meet the control system’s specifications. The uncertain plant can be described as

\[
P(s, \alpha) = \sum_{i=0}^{m} p_i(\alpha) s^i + \sum_{i=0}^{n} q_i(\alpha) s^i
\]

where \( \alpha \in \Omega \subset R^p \) is an uncertain parameter vector. \( \Omega \) is a compact set of parameter variations and may be given as

\[
\Omega = \{ \alpha; \alpha_i \in [\alpha_i, \bar{\alpha}_i], i = 1, \ldots, p \}
\]

where each uncertain parameter \( \alpha_i, (i = 1, \ldots, p) \) varies independently within an interval. Choosing a reference parameter vector, \( \alpha_0 \), yields the nominal plant \( P_0(s) = P(s, \alpha_0) \). Many problems of practical interest can be expressed by the above plant model with parametric uncertainty.

QFT Design Problem

The QFT design problem is to synthesize a pair of strictly proper, rational, stable functions \( G(s) \) and \( F(s) \) such that the following specifications are satisfied while the bandwidth of the controller is kept as low as possible.

- **Robust Stability:** The closed-loop system \( T(s, \alpha) = \frac{L(s, \alpha)}{1 + L(s, \alpha)} \) must be stable \( \forall \alpha \in \Omega \). \( L(s, \alpha) = P(s, \alpha)G(s) \) is the open-loop transfer function.

- **Robust Tracking:** A tracking specification for the reference input is given as

\[
T_f(\omega) \leq |F(i\omega)T(i\omega, \alpha)| \leq T_u(\omega) \quad \omega \in [0, \infty)
\]

where the frequency response bounds, \( T_f(\omega) \) and \( T_u(\omega) \), are specified by the designer.

- **Disturbance Attenuation:** The requirement for disturbance rejection at plant output is expressed as

\[
\max_{\alpha \in \Omega} |T_d(j\omega, \alpha)| \leq M_d(\omega)
\]

where

Electrohydraulic Actuator Modeling

Presentation of Nonlinear Dynamic Equations

A schematic diagram of a hydraulic actuator in contact with the environment is shown in Fig. 1. In this study, the well-known model of manipulator-sensor-environment [23], [24] is coupled with the nonlinear hydraulic actuator dynamics. The sensor with stiffness \( k_s \) and damping \( d_s \) connects the actuator piston, represented by the mass \( m_a \), to the environment. The environment is represented by mass \( m_e \), damping \( d_e \), and stiffness \( k_e \). The dynamic equations are

\[
m_a \ddot{x} = f_u - d_e (\dot{x} - \dot{x}_e) - d_s \dot{x} - k_e (x - x_e)
\]

(1)

\[
m_e \ddot{x}_e = d_s (\dot{x} - \dot{x}_e) - d_e \dot{x}_e + k_e (x - x_e) - k_e x_e
\]

(2)

\[
f = k_e (x - x_e)
\]

(3)
\[ T_o(s,\alpha) = \frac{1}{1 + L(s,\alpha)} \]

is the transfer function from the disturbance to the output and \( M_o(\omega) \) is the magnitude of disturbance rejection.

**QFT Design Procedure**

The design procedure involves the following steps.

1) *Generating the plant templates*: The region in the Nichols chart occupied by the complex values of \( P(j\omega,\alpha) \) at each frequency is called the plant template (see the figure at right). This essentially characterizes the plant uncertainty by capturing the gain and phase variations of the plant at a given frequency. The templates are used to generate bounds on the Nichols chart for the controller design.

2) *Generating performance bounds*: The design specifications outlined above should be translated into certain bounds on the nominal open-loop transfer function, \( L_o(s) = P_o(s) G(s) \), to reveal the trade-offs between performance specifications and robustness at each frequency. These bounds are derived either by moving the plant template between the closed-loop magnitude contours on the Nichols chart or by a computer search. Examples of bounds are shown in the figure. These bounds are used in the next step as a guide for loop shaping the nominal open-loop transfer function.

3) *Loop shaping*: Once the QFT bounds are determined, the nominal loop transfer function, \( L_o(s) \), should be designed, by adding proper poles and zeroes, to yield a stable nominal closed loop, while at the same time satisfying all bounds. A typical plot of \( L_o(s) \) is given in the figure. An optimum \( L_o(s) \) is one that satisfies the bounds and decreases as rapidly as possible with frequency to keep the controller bandwidth small. During this stage, the designer should effect a trade-off between conflicting specifications, controller complexity, and the cost of the feedback in the bandwidth. Once a satisfactory \( L_o(s) \) is arrived at, the controller can be extracted from \( L_o(s) \) by dividing it by the nominal plant transfer function \( P_o(s) \).

4) *Design of prefilter*: Design of a proper \( L_o(s) \) only guarantees that the variation in the closed-loop transfer function, \( T(s,\alpha) \), is less than or equal to that allowed. Therefore, a prefilter is required to bring the response within the upper and lower tolerances, \( T_o(\omega) \) and \( T_o(\omega) \). The prefilter magnitude bound at each frequency can be calculated from the robust tracking specification.

\[
q_i = c_d \omega x_{wp} \sqrt{\frac{2}{\rho}} (p_i - p_o) \quad q_o = c_d \omega x_{wp} \sqrt{\frac{2}{\rho}} (p_o - p_i) \quad (5b)
\]

where \( q_i \) and \( q_o \) represent fluid flows into and out of the valve, respectively. \( c_d \) is the orifice coefficient of discharge, \( \rho \) is the mass density of the fluid, \( p_i \) is the pump pressure, and \( p_o \) is the return (exit) pressure. \( w \) is the area gradient that relates the spool displacement \( x_{wp} \) to the orifice area. Continuity equations for oil flow through the cylinder, neglecting the leakage flow across the actuator’s piston, are

\[
q_i = A_i \frac{dx}{dt} + \frac{1}{\beta} V_i \frac{dp_i}{dt} \quad (6a)
\]

\[
q_o = A_o \frac{dx}{dt} - \frac{1}{\beta} V_o \frac{dp_o}{dt} \quad (6b)
\]
Here $\beta$ is the effective bulk modulus of the hydraulic fluid, and $V_i$ and $V_o$ are the volumes of fluid trapped at the sides of the actuator. They can be expressed as functions of actuator displacement $x$.

$$V_i(x) = V_i + xA_i \quad (7a)$$

$$V_o(x) = V_o - xA_o \quad (7b)$$

where $V_i$ and $V_o$ are the initial volumes trapped in the blind and rod sides of the actuator. The relationship of the spool displacement, $x_{sp}$, and the input voltage, $u$, to the proportional valve can be expressed as a first-order system that is accurate enough to model low-cost proportional valves [26]

$$u = \frac{\tau}{k_{wp}} \frac{dx_{wp}}{dt} + \frac{1}{k_{wp}} x_{wp} \quad (8)$$

where $\tau$ and $k_{wp}$ are gains describing the valve dynamics. Equations (1)-(8) express the relationship between the contact force, $f$, and the input control voltage, $u$.

**Hydraulic actuators are advantageous because of their high force-to-weight ratio and fast response time.**

$$V_i(x) = V_i + xA_i \quad (7a)$$

$$V_o(x) = V_o - xA_o \quad (7b)$$

They are given as follows:

for extension ($x_{wp} \geq 0$)

$$K_i' = c_{wi} \frac{g}{\rho} (p_i - p_o)$$

$$K_i^o = c_{wo} \frac{g}{\rho} (p_o - p_i)$$

$$K_p^i = \frac{c_{wi} x_{wp}}{\sqrt{2}(p_i - p_o)}$$

$$K_p^o = \frac{c_{wo} x_{wp}}{\sqrt{2}(p_o - p_i)}$$

As can be seen, $K_i'$ ($K_i^o$) and $K_p^i$ ($K_p^o$) are load- and pressure-dependent variables. Hence, in this study, they are considered uncertain but bounded parameters. Assuming small piston displacements within the vicinity of the midstroke, the following approximation is made:

$$\frac{V_i(x)}{\beta} = \frac{V_o(x)}{\beta} = \frac{1}{\beta} \left( \frac{V_i + V_o}{2} \right) = C. \quad (10)$$

Thus, (6) can be written in the Laplace domain as

$$Q_i = A_i s X + C s P_i \quad (11a)$$

$$Q_o = A_o s X - C s P_o \quad (11b)$$

Substituting (9) into (11) and rearranging for line pressures, we have

$$P_i = -\frac{A_i s}{C s + K_p^i} X + \frac{K_i'}{C s + K_p^i} x_{wp} \quad (12a)$$

$$P_o = \frac{A_o s}{C s + K_p^o} X - \frac{K_o'}{C s + K_p^o} x_{wp} \quad (12b)$$
As shown below, where the transfer function describing the relation between contact force and spool displacement is derived as:

\[ F_a = \frac{K_p A_b}{Cs + K_p} \]

Incorporating (12) into (4) we have

\[ F_a = K_p A_b + K_p A_b \frac{X_p}{X_s} = K_s A_s + K_s A_s \frac{X_p}{X_s} \]

Equations (1)-(3) are now combined to form the following transfer functions:

\[ F(s) = \frac{k_i}{s} \left( m_i s^2 + d_i s + k_i \right) \]

\[ X(s) = \frac{k_i}{s} \left( m_i s^2 + d_i s + k_i \right) \]

Substituting these equations into (13) and rearranging it, the transfer function describing the relation between contact force and spool displacement is derived as:

\[ \frac{F(s)}{F(s)} = \frac{K_i (A_i + A_i)(m_i s^2 + d_i s + k_i)}{(K_p + Cs)(m_i s^2 + d_i s + k_i) + (A_i s + A_i s)} \]

Further, we assume the dynamics of the environment is dominated by a pure stiffness, \( k_e \). This type of environment has enjoyed popularity among many researchers [27]. The system transfer function is finally written in its simplest form as

\[ \frac{F(s)}{X_p(s)} = \frac{k_i}{s} \left( m_i s^2 + d_i s + k_i \right) \left( K_p A_b + K_p A_b \frac{X_p}{X_s} \right) \]

\[ \frac{F(s)}{X_p(s)} = \frac{k_i}{s} \left( m_i s^2 + d_i s + k_i \right) \left( K_i (A_i + A_i)(m_i s^2 + d_i s + k_i) + (A_i s + A_i s) \right) \]

Table 1. Operating values and parameter ranges pertaining to the linear transfer function.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal Value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_e )</td>
<td>75 (kN/m)</td>
<td>50-100</td>
</tr>
<tr>
<td>( K_i )</td>
<td>0.375 (m²/pa.s)</td>
<td>0.25-0.5</td>
</tr>
<tr>
<td>( K_p )</td>
<td>2.5(10⁻¹²) (m³/s)</td>
<td>0-5(10⁻¹²)</td>
</tr>
<tr>
<td>( C )</td>
<td>1.5(10⁻¹¹) (m³/pa)</td>
<td>1(10⁻¹¹)⁻3(10⁻¹¹)</td>
</tr>
<tr>
<td>( d )</td>
<td>700 (N/m/s)</td>
<td>600-800</td>
</tr>
<tr>
<td>( m_e )</td>
<td>20 (kg)</td>
<td>19.9-20.1</td>
</tr>
<tr>
<td>( A_i )</td>
<td>0.00203 (m²)</td>
<td>0.00193-0.00213</td>
</tr>
<tr>
<td>( A_o )</td>
<td>0.00152 (m²)</td>
<td>0.00144-0.00160</td>
</tr>
<tr>
<td>( k_{wp} )</td>
<td>0.0012 (m/V)</td>
<td>0.0011-0.0013</td>
</tr>
<tr>
<td>( \tau )</td>
<td>35 (ms)</td>
<td>30-40</td>
</tr>
</tbody>
</table>

Including the dynamics of the valve from (8), the transfer function between measured contact force, \( F(s) \), and control voltage, \( U(s) \), is written as:

\[ \frac{F(s)}{U(s)} = \frac{k_i}{(s + 1)} \left( \frac{K_i (A_i + A_i)(m_i s^2 + d_i s + k_i)}{(K_p + Cs)(m_i s^2 + d_i s + k_i) + (A_i s + A_i s)} \right) \]

Equation (20) is now considered as a parametrically uncertain system. For example, the uncertainty ranges in \( K_i \) and \( K_p \) reflect variations in the operating point, supply pressure, and, in part, the orifice area gradient. The variations in the environmental stiffness and damping of the system are included in parameters \( k_e \) and \( d \), respectively. Furthermore, the uncertainty in \( C \) represents the changes in the fluid bulk modules and the volumes of fluid trapped at the sides of the actuator. The uncertainty in the valve characteristic is captured in variations of \( \tau \) and \( k_{wp} \). All these parameters are known to have an effect on the system’s stability.

Table 1 lists the nominal values of all parameters in (20) and their corresponding uncertainty ranges. These values, which are representative of the hydraulic actuator under investigation (see the “Experimental Setup” section), were obtained from available references (including manufacturer specifications), direct measurements of some geometric parameters, and indirect identification based on experimental observation of the responses [26]. These uncertain parameter...
ters are grouped into a vector, denoted as $\alpha$. Then the system’s open-loop transfer function can be defined as

$$P(s,\alpha) = \frac{F(s)}{U(s)}$$  \hspace{1cm} (21)

The plant evaluated at its nominal operating point is referred to here as the nominal plant and is shown as

$$P_0(s) = P(s,\alpha_0)$$  \hspace{1cm} (22)

where $\alpha_0$ is the vector containing all nominal values of the plant transfer function.

**Controller Synthesis**

The goal of this section is to design a robust force controller for the hydraulic actuator that is represented by the uncertain transfer function (21). A typical two-degree-of-freedom feedback system configuration in QFT is shown in Fig. 2. A strictly proper controller, $G(s)$, and a strictly proper prefilter, $F(s)$, are to be designed such that the following conditions are satisfied (readers are referred to [28] for a description of the QFT design procedure):

\begin{itemize}
  \item [i)] Closed-loop robust stability. The associated QFT robustness constraint in terms of the nominal loop transfer function, $L_n(s) = P_0(s)G(s)$, is given by [28]:

  $$\left| \frac{L_n(i\omega)}{P_0(i\omega)} \right| \leq M = 1.4 \hspace{1cm} \forall \omega \in [0, \infty)$$  \hspace{1cm} (23)

  which implies an approximately 3 dB gain margin for the closed-loop system.
\end{itemize}
ii) **Robust reference input.** To meet tracking performance requirements, the controller should satisfy the following inequality:

\[ \|p(i\omega)\|_{\infty} \leq \left( \frac{L_0(i\omega)}{\rho(i\omega,\alpha)} + L_{\rho}(i\omega) \right) \leq \|p(i\omega)\|_{\infty} \quad \forall \omega \in [0, \infty) \]

(24)

where the upper and lower tracking bounds are defined as

\[ T_u(s) = \left( \frac{s}{2.8} \right) + 1 \]

\[ T_l(s) = \left( \frac{s}{4.8} \right) + 1 \left( \frac{s}{8} + 1 \right) \left( \frac{s^2}{80} + 1 \right) \left( \frac{9.6s}{50} + 1 \right) \]

(25a)

(25b)

These bounds are built from the time domain figures of merit for step responses such as peak overshoot, peak time, and settling time [14]. In this work, the desired lower tracking bound, \( T_l(s) \), is built to have an overdamped response with \( \approx 1 \) s settling time. For this purpose, a model with a real pole at \( s = -4.8 \) and a pair of complex poles are chosen. The real pole must be more dominant than the complex poles [14]. Moreover, a high-frequency pole at \( s = -80 \) is inserted in \( T_l(s) \), which does not affect the desired performance specification but widens the range between \( T_u(s) \) and \( T_l(s) \) in the high-frequency band. The figures of merit for upper bound, \( T_u(s) \), are a 5% peak overshoot and \( \approx 1 \) s settling time. \( T_u(s) \) is selected with three real poles and a zero. The zero is closer to the origin than the poles to have an underdamped response. The frequency and time domain plots of these bounds are shown in Fig. 3(a) and (b), respectively.

iii) **Closed-loop disturbance attenuation (sensitivity reduction).** For disturbance rejection at the plant output, an upper tolerance is imposed on the sensitivity function. Here we consider only a constant upper bound to limit the peak value of disturbance amplification as follows:

\[ M_{\rho}(\omega) = 1.2 \quad \forall \omega \in [0, 0.1] \]

(26)

The above design specifications impose constraints on the allowable loop gain, \( |L_0(\omega)| \). These constraints can be shown on a Nichols chart as a curve boundary at each design frequency. Once the bounds are derived, the nominal open-loop transfer function, \( L_0 \), should be determined such that it lies on or above the corresponding bounds at every frequency. This can be done using a procedure that is called loop shaping in QFT. Finally, the controller can be extracted from \( L_0 \) by dividing by the nominal plant transfer function, \( P_0 \).

Design frequencies are selected as \( \omega = \{0.01, 0.05, 0.1, 0.5, 1, 5, 10, 50, 70, 100\} \) rad/s. The bounds generated by constraints (23), (24), and (26) are shown in Fig. 4(a) for the uncompensated plant. The nominal open-loop transfer func-

![Figure 5](image-url)
Maintaining a low-order controller was an important factor in the design process.

stabilization, $L_0$, must be shaped to satisfy all these bounds at each frequency. For low frequencies, $L_0$ should lie on or above the bounds to satisfy the constraints. However, for higher frequencies in the range, e.g., $\omega = 50$, 70, and 100 rad/s, the constraints generate closed boundaries which $L_0$ should not enter. For the industrial hydraulic actuator under investigation, the valve dead band produces a steady-state error in the system response in the absence of an integrator. For a zero steady-state error, $L_0$ must contain an integrator. Thus, a possible loop can be obtained by cascading an integrator. Further, a gain of 0.0065 is required to bring the open-loop transfer function within the specification bounds. Two zeroes should be added to the structure of the controller to satisfy the bounds and to force the nominal loop, $L_0$, to the right-hand side of the closed boundaries. After several iterations the controller zeroes were chosen at $s = -4$ and $s = -40$. At this stage, $L_0$ closely follows the bounds up to $\omega = 70$ rad/s.

The next step is to add two high-frequency poles to allow for a quick descend of the nominal loop to decrease the cost of feedback. Fig. 4(b) shows the final loop shaping of the system. The controller that satisfies the specifications is

$$G(s) = \frac{0.0065 \left( \frac{s}{4} + 1 \right) \left( \frac{s}{40} + 1 \right)}{s \left( \frac{s}{100} + 1 \right) \left( \frac{s}{300} + 1 \right)}.$$ (27)

To satisfy the tracking specification, a prefilter, $F(s)$, is required to place the closed-loop frequency response in the specified range, between $T_u(s)$ and $T_i(s)$. Fig 5(a) shows the frequency response of the closed-loop system without a prefilter where the designed controller (27) only guarantees that the variation in the system’s magnitude will be less than or equal to that allowed. The prefilter is synthesized by calculating its magnitude bound at each frequency. For example, in Fig. 5(a), consider the frequency response of the control system at $\omega = 50$ rad/s. The maximum frequency variation, $\delta$, is about 17 dB = $-25$ dB, which is less than the amount allowed for this frequency, i.e., $\delta_{\omega} = 25$ dB = $|T_u(50)| - |T_i(50)| = -30$ dB. The acceptable magnitude of the prefilter at this frequency is then found to be $-30 - (55) \leq |F(50)| \leq -30 - (55)$. By applying the above procedure for the design frequency range, a suitable prefilter is found to be

$$F(s) = \frac{\left( \frac{s}{150} + 1 \right)}{\left( \frac{s}{55} + 1 \right) \left( \frac{s}{9} + 1 \right)}.$$ (28)

The closed-loop Bode plots of the plant with prefilter are given in Fig. 5(b). Note that all the tracking specifications are met in the frequency domain. The corresponding closed-loop simulated time responses to a step input of 1000 N are shown in Fig. 5(c). As can be seen, for all cases pertaining to extreme parts of the operating envelope, the specifications are satisfied.

**Experiments**

**Experimental Setup**

The test station consists of a hydraulic unit, a 486/66-based PC equipped with a Metrabyte M5312 quadrature incremental encoder card, and a DAS-16 analog-to-digital (A/D) conversion card (Fig. 6). The pump provides constant operational supply pressure up to $\approx$ 1000 psi. The hydraulic valve is a low-cost closed-center four-way proportional valve. The positioning of the valve spool is based on the pulse-width modulation principle. A spring is used to rep-
resent the environment (Fig. 7); replacing the spring changes the stiffness of the environment. A force transducer with a capacity of 1000 lb (4.5 kN) is inserted between the environment and the piston rod. The 12-bit A/D conversion allows a force resolution of 10 N when the full range is used. Three pressure transmitters read pump, supply line, and return line pressures with ±1% accuracy. An incremental encoder with sensing resolution of 0.06 mm reads the displacement of the cylinder piston. The control signal, generated by the control algorithm, is converted to an analog signal by the A/D card and then transmitted to the hydraulic valve amplifier. The low-cost valve operates within the range ±1.8 V and has a dead band of ±10% (±0.15 V) within which the actuator does not move. No attempt was made to eliminate or bypass these nonideal effects due to the intended application.

Results

The controller described by transfer function (27) was discretized and implemented on the experimental test stand. The sampling frequency for the controller was 200 Hz. Several experiments were performed to study the effects of variations of environmental stiffness, supply pressure, and force setpoint. First, the variation of environmental stiffness was studied by using different springs. Two different springs with stiffnesses of 50 and 100 kN/m were used for this purpose. The results are shown in Fig. 8. As can be seen, the controller is capable of handling the changes in environmental stiffness and the steady-state errors are small. With reference to Fig. 8(a), responses exhibit initial delays, mainly due to the dead band (±10%) and dry friction in the low-cost valve used in this experiment. Consequently, the responses do not precisely fit within the design specification bounds shown with dotted lines. Indeed, the effect of valve dead band was not incorporated into the QFT design procedure and was only handled by cascading an integrator in the controller. The actuator displacement and the control signal for such a trial are shown in Fig. 8(b) and (c), respectively. The control signal is smooth, but it contains high-frequency oscillations that originated from the noise in the force sensor. Fig. 9 compares the test results for two different reference forces (500 N and 1000 N) and with similar environmental stiffness (50 kN/m). As shown in this figure, despite changing the loading condition, the system’s rise time did not change considerably. The ability of the controller to cope with pump pressure variations was also tested. Typical results are shown in Fig. 10, where the pump pressure was varied 100%. As can be seen, the control effort is reduced for the higher pump pressure. Finally, a test was arranged to investigate the repeatability of the controller and the effect of long-term system operation on the response. A 1000-N step response was performed once and then again after the machine had been in continuous operation for more than one hour. As shown in Fig. 11, there is no noticeable difference between the two responses.

Conclusions

This article has described the application of the QFT technique to the development of a force controller for hydraulic actuators. A linear parametrically uncertain fourth-order model was developed to represent the relation between the control signal and the force acting on the environment.

Figure 8. Step responses with different environmental stiffnesses (experiment): (a) force, (b) control signal, and (c) displacement.
uncertainty was quantified within the linear model to compensate for time-varying dynamics and to allow for variations in the environmental stiffness, hydraulic fluid flow and pressure gains, actuator compliance, and many other parameters specific to the hydraulic actuator components. A robust controller was then designed that, along with a prefilter, maintains a satisfactory force control performance against the environment despite a wide range of uncertainty. Maintaining a low-order controller was an important factor in the design process to ensure easy implementation and demonstrate its suitability for industrial applications.

The QFT controller designed here was implemented successfully on an industrial hydraulic actuator equipped with a low-cost proportional valve. The transparency of the design process facilitated the quick adjustment of the controller for experimental implementation. Several tests were performed to show the force-regulating ability of the developed controller. In particular, the experimental results demonstrated the robustness of the QFT controller for up to 100% variations in environmental stiffness, supply pressure, and reference force. Good performance was also obtained despite significant variations in actuator dynamics. The single-loop fixed-gain controller was easy to implement, required very little computational effort, needed no online tuning or gain scheduling, and resulted in good performance in both transient and steady-state periods. Knowledge of the lower and upper bounds of uncertain parameters was the only requirement for the controller design.

In summary, the results of this work, which represent a contribution to the published work on the application of QFT, provide insight into the potential and effectiveness of the technique for the design of robust force controllers for hydraulic actuators. In particular, we have shown that it is feasible to employ a single fixed-gain force controller for a
hydraulic actuator, and that control performance can be insensitive to structured plant variation. It also became apparent that the uncertain linear fourth-order model adequately characterizes the major features of the class of hydraulic systems under investigation.

In future work, we plan to use the QFT method to design automatic force controllers for heavy-duty hydraulic equipment such as excavators. These machines strongly interact with environments, and accurate models for the interactions are difficult to obtain. Currently, we are investigating the feasibility of assuming the environmental force as a combination of disturbance and unstructured uncertainty to be handled by the QFT controller.

References


Nariman Sepahi received the B.Sc. degree from Sharif University of Technology, Iran, and the M.Sc. degree from Isfahan University of Technology, Iran, in 1994 and 1997, respectively, both in mechanical engineering. Since January 1998, he has been with the Experimental Robotics and Teleoperation Laboratory, University of Manitoba, Canada, where he has held teaching and research assistantship positions. His current research interests include robust control and control of discontinuous and nonlinear systems, with applications to robotics and hydraulic systems. He received several awards and a fellowship from the University of Manitoba during 1999-2001.

Nariman Sepahi joined the Department of Mechanical and Industrial Engineering at the University of Manitoba in 1991, where he is currently a Professor. He received an M.Sc. in computer-aided process planning in 1986 and a Ph.D. in robotics and controls in 1990, both from the University of British Columbia. His areas of interest are mathematical modeling and real-time simulation, robust control design, and fluid power. An area of research interest is the automation of heavy-duty industrial (forest and mining) machinery using robotics and advanced control technology. He is a member of the American Society of Mechanical Engineers and the IEEE.