Fault isolation for linear non-minimum phase systems using dynamically extended observers

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Abstract—This article is devoted to the design of fault isolation observers (FIOs). Their purpose is to detect and isolate faults occurring in linear systems by means of only a single, specifically parameterized observer. While parameterizations for such observers have been proposed for minimum phase systems, we generalize the existing results to linear systems without such restrictions. To achieve stable fault isolation, we introduce virtual system extensions, which enable stable isolation despite right half-plane zeros. These virtual augmentations are then realized by dynamically extended fault isolation observers (DFIOs). Apart from that, the proposed design allows to isolate both actuator and sensor faults. The applicability of the proposed approach is shown in both simulations and lab experiments with a gantry crane.

I. INTRODUCTION

Fault diagnosis has been attracting a lot of attention both in the scientific community and industrial applications over the past decades [6], [7], which is due to increasing complexity of systems and higher demands on reliability. Much progress can be observed in the development of model-based approaches [2], [3], which incorporate knowledge about the system dynamics into the process of fault diagnosis.

Often, observer-based approaches are used to generate residuals, which act as fault indicators. Many recent results focus on robustness issues with respect to parametric uncertainties and/or exogeneous disturbances [10], [14], [28], [30], [31].

However, these approaches are devoted to mere fault detection. A second important stage is to isolate faults, i.e., determine the type and location of possible multiple faults in the system. While this can be achieved by employing a bank of specifically designed fault detection observers [5], fault isolation observers (FIOs) where introduced in [11] aiming at fault isolation by using only a single observer. On the one hand, this obviously results in a simpler structure of the fault diagnosis system and a decreased implementation effort. On the other hand, robustness is not easy to achieve and the approach is limited to statically isolable and minimum phase systems. While the former has been dealt with e.g. in [1], [9], [26], we focus on a part of the latter problem in this paper. While systems with singular fault detectability matrix are covered in [25], we turn our attention to non-minimum phase systems in this paper. The main contribution is the development of stable FIOs for such systems.

To this end, we infer duality to non-interacting control, which dates back to [4]. In [16], it is shown that non-minimum phase systems that do not allow stable decoupling by static state feedback can be stably decoupled by dynamic state feedback. We extend these results and modify them for the application in observer-based fault isolation. A key tool is the parametric approach to eigenstructure assignment ([12], [24]), which has been shown to be applicable to observer design [13], [23] and especially FIO design [26].

Furthermore, we show that the additional dynamics needed can be implemented by dynamically extended observers, which were introduced by [21]. A special form of this observer structure was used in fault diagnosis in [19] and more recently in the general form in [22], [27], [29]. Our results demonstrate another advantage of the additional dynamics by extending the class of systems that can be fault isolated. In our context, we refer to such observers as dynamically extended fault isolation observers (DFIOs).

Our approach is directly applicable for both actuator and sensor faults. Therefore the indirect treatment of sensor faults as pseudo-actuator faults [20] is omitted resulting in a decreased order of the fault isolation system.

Hence, the paper is structured as follows. In Section II, we summarize some notational aspects and give a proper problem statement. Furthermore, we briefly recall the parametric design of static FIOs in Section III. Section IV is devoted to the observer-based fault isolation for non-minimum-phase systems and DFIO-realizations of such fault isolation systems. We present some simulation and measurement results of lab experiments with a gantry crane to demonstrate the applicability of the presented approach in Section V before a conclusion is given.

II. PRELIMINARIES

A. Notation and mathematical background

The identity matrix of order $n$ is written as $I_n$, while $0$ denotes a matrix of zeros of appropriate dimensions. To improve readability, we write $0_{n	imes m}$ in some equations to visualize the dimensions of the zero matrix. For a complex matrix $Q \in \mathbb{C}^{n\times m}$, the complex conjugate is denoted by $Q^*$, while the Kronecker product of two matrices $P$ and $Q$ is written as $P \otimes Q$. The $i$-th column of a matrix $P$ is written as $p_i$, while $p^T_j$ stands for the $j$-th row. We further introduce $\phi_i = [0 \cdots 0 1 0 \cdots 0]^T$ with element 1 in the $i$-th row and a diagonal matrix $P \in \mathbb{R}^{n\times n}$ is written as $P = \text{diag}(p_{11}, \ldots, p_{nn})$, where $p_{ii}$ are the diagonal elements. The degree of a polynomial $z(s)$ is written as $\deg(z(s))$, a system $\dot{x} = Ax + E_0f$, $y = Cx + E_sf$ is
abbreviated by \((A, E_a, C, E_s)\), and its Rosenbrock matrix is written as
\[
\Sigma(s) = \begin{bmatrix}
sI_n - A & E_a \\
-C & E_s
\end{bmatrix}.
\] (1)

Standard Luenberger-type observers [17] use static feedback of the difference \(y - C\dot{x}\) between plant and observer output to tune the observer error dynamics,
\[
\dot{x} = A\dot{x} + Bu + L(y - C\dot{x}).
\] (2)

This concept is extended in [21], where dynamic feedback is used in the observer leading to dynamically extended observers of the form
\[
\dot{x} = A\dot{x} + Bu + v, \quad v = L\dot{x} + L_p(y - C\dot{x}), \quad \dot{\xi} = \Phi\xi + \Gamma(y - C\dot{x})
\] (3a)\,(3b)\,(3c)

with the internal observer state \(\xi \in \mathbb{R}^\nu\) and matrices \(L_p, L, \Phi, \Gamma\) of appropriate dimensions. In this paper, we employ dynamically extended observers for fault isolation, where residuals \(r \in \mathbb{R}^{n_f}\) are generated by
\[
r = V_1\xi + V_2(y - C\dot{x}).
\] (3d)

Dynamically extended observers are abbreviated as dynamic observers in the remainder of this paper while Luenberger observers are referred to as static observers.

**B. Problem statement**

We consider linear systems described by
\[
\begin{align*}
\dot{x} &= Ax + Bu + E_af, \quad (4a) \\
y &= Cx + E_sf.
\end{align*}
\] (4b)

with \(x \in \mathbb{R}^n, u \in \mathbb{R}^{nu}, f \in \mathbb{R}^{n_f}, y \in \mathbb{R}^{nv}\) and matrices of appropriate dimensions. The actuator and sensor faults \(f\) are to be detected and isolated by means of a dynamically extended observer of the form (3). Thus, the transfer matrix relating faults and residuals,
\[
G_{rf}(s) = \text{diag}(g_{r_1f_1}(s), \ldots, g_{r_nf_{nf}}(s))
\] (5)
is to be rendered diagonal by properly parameterizing (3). Therein, \(g_{r_if}(s)\) is the transfer function relating fault \(f_i\) and residual \(r_i\). Due to the strict diagonal structure of \(G_{rf}(s)\), even simultaneously occurring faults can be isolated. A dynamically extended observer with this isolation property is called dynamically extended fault isolation observer (DFIO). Note that we explicitly consider sensor faults in our approach. In contrast to [20], they do not have to be recast as pseudo-actuator faults. Thus, no knowledge on the fault dynamics is needed achieving a more transparent design procedure with less design parameters.

The fault detectability indices \(\delta_i\) with \(i = 1, \ldots, n_f\) are defined as
\[
\delta_i = \begin{cases} 
0, & \epsilon_s = 0, \\
\min \{k \geq 1: CA^{k-1}e_a = 0\}, & \epsilon_s \neq 0
\end{cases} \] (6)

and the fault detectability matrix \(D^* \in \mathbb{R}^{n_x \times n_f}\) is written accordingly as
\[
D^* = \begin{bmatrix} d_{11}^* & \cdots & d_{nf}^* \end{bmatrix},
\] (7a)
\[
d_i^* = \begin{cases} e_s, & \delta_i = 0, \\
CA^{k-1}e_a, & \delta_i \geq 1.
\end{cases} \] (7b)

Furthermore, we call \(\delta = \sum_{i=1}^{n_f} \delta_i\) the total fault index and the number of closed right half-plane (cRHP) zeros, i.e., with \(\Re(\eta) \geq 0\), is denoted as \(\mu_{c\text{RHP}}\).

**Assumption 1:** The equation
\[
\text{rank}(\Sigma(\eta)) = n + n_f
\]
holds for almost all \(\eta \in \mathbb{C}\), i.e., the system \((A, E_a, C, E_s)\) only has a finite number of invariant zeros.

**Assumption 2:** \(\eta_0 = 0\) is not an invariant zero of \((A, E_a, C, E_s)\).

**Assumption 3:** The pair \((A, C)\) is observable.

**Assumption 4:** No invariant zero of \((A, E_a, C, E_s)\) is an eigenvalue of \(A\).

**Assumption 5:** \(D^*\) is regular and the system \((A, E_a, C, E_s)\) is square, i.e., \(n_x = n_f\).

Note that Assumption 4 imposes no restriction since we can always shift the eigenvalues of \(A\) arbitrarily due to Assumption 3 prior to the DFIO design.

With this definitions and assumptions, our approach is visualized in Fig. 1. The system is repeatedly augmented for each cRHP zero (cf. Section IV-A), which we will show to enable the design of a stable static FIO for the augmented, virtual system (cf. Section III). This FIO can then be implemented by properly parameterizing a DFIO for the original system (cf. Section IV-B).

Note that we explicitly do not consider any parametric uncertainties or disturbances in the model (4) since we will focus on the design of dynamic observers for fault isolation in non-minimum phase systems. However, we will reduce the problem of designing DFIOs for non-minimum phase systems to the known problem of parameterizing static FIOs for augmented systems. Thus, robustness issues can be addressed by the standard procedures available for minimum phase systems. Thereby, it is possible to increase robustness both with respect to parametric uncertainties as well as exogeneous disturbances e.g. by using a model matching.
approach ([18], [31]) for the augmented system. We omit
the details here to keep the focus on our main issue and due
to space restrictions but emphasize that standard methods can
be employed to increase robustness.

III. PARAMETRIC FIO DESIGN

Since we will reduce the problem of designing DFIOS to
constructing static FIOs for augmented systems, we briefly
review the parametric approach to designing such fault
isolation observers.

Using an observer (2), residual generation \( r = V(y - C\hat{x}) \) and introducing \( e = x - \hat{x} \), the transfer matrix \( G_{rf}(s) \) relating faults and residuals is obtained as

\[
G_{rf}(s) = V \left( C(sI_n - (A + LC))^{-1}(E_a - LE_a) + E_s \right). \tag{8}
\]

Obviously, a necessary condition to satisfy (5) is \( G_{rf}(0) = \text{diag}(g_{r_1f}(0), \ldots, g_{r_nf}(0)) = S \), where the static gains \( g_{r_jf}(0) \neq 0 \) may be arbitrarily chosen. Since \( S \in \mathbb{R}^{n_f \times n_f} \) is a full rank matrix, the condition

\[
G_{rf}(0) = V \left[ C(A - LC)^{-1}(E_a - LE_a) + E_s \right] = S \tag{9}
\]

implies that 0 must not be an eigenvalue of \( A - LC \) and that both \( V \) and \( M \) have to be regular matrices. Assumption 2 is necessary to guarantee regularity of \( M \), which can be proven by contradiction. To this end, assume \( \eta = 0 \). Multiplying \( \Sigma(\eta = 0) \) from the left with two specific regular matrices gives

\[
\begin{bmatrix}
I_n \\
-C(A - LC)^{-1}I_n \\
0
\end{bmatrix}
\begin{bmatrix}
I_n & -L \\
0 & I_{n_f}
\end{bmatrix}
\begin{bmatrix}
-A & E_a \\
C & E_s
\end{bmatrix} =
\begin{bmatrix}
-A - LC \\
-C(A - LC)^{-1}E_a - LE_a \\
0
\end{bmatrix}.
\tag{10}
\]

Due to the block diagonal structure, calculating the determin-
ant leads to

\[
det(A - LC) \cdot \det(M) \neq 0, \tag{11}
\]

if \( \eta = 0 \) is an invariant zero. Since 0 must not be eigenvalue of \( A - LC \), this implies singularity of \( M \) and hence perfect fault isolation in the sense of (5) is not possible.

Based on the above reasoning, the post-filter matrix \( V \) is obtained by

\[
V = SM^{-1} \tag{12}
\]

once \( L \) is found. To design the observer gain \( L \), the parametric approach introduced in [23] and employed for fault isolation in [26] can be used. In this paper, the results are extended to deal with sensor faults while the proof is omitted due to space restrictions. The observer gain can be written as

\[
L = -V R P, \tag{13a}
\]

\[
V R = [u_R^T, \ldots, u_R^{T-1}]^{-1} \begin{bmatrix}
p_R^T C(\lambda_{R_1} I_n - A)^{-1} \\
p_R^T C(\lambda_{R_2} I_n - A)^{-1} \\
\vdots \\
p_R^T C(\lambda_{R_n} I_n - A)^{-1}
\end{bmatrix}. \tag{13b}
\]

Therein, \( p_R^T \) are called parameter vectors and \( u_R^T \) is the left-eigenvector corresponding to the \( k \)-th eigenvalue \( \lambda_{R_k} \) of \( A - LC \). To simplify notation, we require no eigenvalue \( \lambda_{R_k} \) to be an eigenvalue of \( A \) and \( \lambda_{R_k} \neq \lambda_{R_l} \) for \( i, j = 1, \ldots, n \) in this paper. Note however, that it is possible to extend the parametric approach to both cases ([12], [24]). As shown in [26], the first \( \delta \) parameter vectors are specified by

\[
p_R^T = \phi_j^T \left( C(\lambda_{R_k} I_n - A)^{-1}E_a + E_s \right)^{-1}, \tag{14}
\]

with \( j = 1, \ldots, n_f, i = 1, \ldots, \delta, \) and \( k = i + \sum_{l=1}^{i-1} \delta_l \).

Thus, an eigenvalue \( \lambda_{R_k} \) is exclusively assigned to the \( j \)-th column of \( G_{rf}(s) \). This can be verified by calculating the partial fractional expansion of \( G_{rf}(s) \).

If the remaining \( n - \delta \) eigenvalues are assigned to a specific column of \( G_{rf}(s) \) they will in general appear in more than one row hindering perfect fault isolation as demanded by (5). While the post-filter matrix \( V \) can be employed to satisfy a diagonal structure of \( G_{rf}(s) \) for \( s = 0 \), this can in general not be achieved for all \( s \in \mathbb{C} \). In order to render the remaining eigenvalues not appearing in \( G_{rf}(s) \) and thus achieve perfect fault isolation, it can be shown that

\[
[w_R^T p_R^T] \cdot \Sigma(\lambda_{R_k}) = 0 \quad \forall k = \delta + 1, \ldots, n_f \tag{15}
\]

has to be fulfilled (26). However, in square systems this equation only has a non-trivial solution if \( \lambda_{R_k} \) is selected to be an invariant zero of (4). For invariant zeros in the open left complex half-plane, this does not cause any stability problems. But for cRHP zeros, i.e. in non-minimum phase systems, this results in unstable eigenvalues \( \lambda_{R_k} \).

IV. DFIOS FOR NON-MINIMUM PHASE SYSTEMS

A. General design procedure

In this section, we deal with statically isolable systems that are non-minimum phase but have a regular \( D^* \). As shown in Section III, it is not possible to design stable FIOs for such systems in general. The cRHP zeros have to be compensated by unstable eigenvalues to achieve isolation. However, as pointed out in [8] and [16] for the dual problem of non-interacting control, the invariant zeros do not have to be compensated if they are non-interconnecting zeros. For such zeros, poles can be assigned to specific columns of \( G_{rf}(s) \) without appearing in any of the off-diagonal elements of \( G_{rf}(s) \). Here, we define such zeros for the problem of fault isolation as follows.

Definition 1: Given a system (4) fulfilling Assumptions 1–5. Let \( \eta \) be an invariant zero of (4), i.e.,

\[
\text{rank}(\Sigma(\eta)) < n + n_f.
\]

We call \( \eta \) a non-interconnecting zero, if

\[
\begin{bmatrix}
z_x \\
z_f
\end{bmatrix} = 0, \tag{16a}
\]

\[
z_f = \phi_j = [0 \ldots 0 1 0 \ldots 0]^T, \tag{16b}
\]

i.e., \( z_x \in \mathbb{R}^n \) is an arbitrary vector and \( z_f \in \mathbb{R}^{n_f} \) contains exactly one non-zero element in an arbitrary row \( j \).
With this definition, we can reformulate a theorem given in [15] for fault isolation and extend it to the case of possible sensor faults.

**Theorem 1**: If \( \eta \) is a non-interconnecting zero of \((A, E_s, C, E_s)\), it does not have to be specified as an observer eigenvalue \( \lambda_{R_k} \) in order to achieve a diagonal structure of \( G_{r_f}(s) \).

**Proof**: We multiply (16a) from the left with two regular matrices, namely

\[
\Phi(\eta) \cdot \Psi \cdot \Sigma(\eta) \cdot \begin{bmatrix} z_x \\ z_f \end{bmatrix} = 0, \quad (17)
\]

with

\[
\Phi(\eta) = \begin{bmatrix} I_n \\ VC(\eta I_n - (A - LC))^{-1} \\ 0 \end{bmatrix}, \quad (18a)
\]

\[
\Psi = \begin{bmatrix} I_n \\ -L \\ 0 \end{bmatrix}, \quad (18b)
\]

Therein, \( \Psi \) is regular due to the regularity of \( V \) and \( \Phi(\eta) \) exists if no eigenvalue of \( A - LC \) is selected to be equal to \( \eta \). Evaluating (17) results in (19), which is given on the bottom of this page and we conclude

\[
G_{r_f}(\eta) \cdot z_f = 0. \quad (20)
\]

Without loss of generality \( z_f = \phi_j \) can be written if \( z_f \) contains exactly one non-zero element in the \( j \)-th row and thus \( g_{r_f}(\eta) = 0 \) holds for all \( i = 1, \ldots, n_f \). Due to \( g_{r_f}(\eta) \) being \( 0 \), assigning \( \delta_j + 1 \) poles to the \( j \)-th column of \( G_{r_f}(s) \) results in

\[
g_{r_f}(s) = \frac{z_{j,0}(s - \eta)}{(s - \lambda_{j,1}) \cdots (s - \lambda_{j,\delta_j+1})} = \frac{z_{jj}(s)}{n_{jj}(s)}, \quad (21)
\]

Since the postfilter \( V \) is designed to guarantee \( G_{r_f}(0) = \text{diag} \left( g_{r_f,1}(0), \ldots, g_{r_f,n_f}(0) \right) \) as presented in Section III, we furthermore know \( g_{r_f}(0) = 0 \) for all \( i \neq j \). The conditions \( g_{r_f}(\eta) = g_{r_f}(0) = 0 \) can only be satisfied if \( g_{r_f}(s) = 0 \forall s \in C \), which can be shown by contradiction as follows. To fulfill the conditions,

\[
g_{r_f}(s) = \frac{z_{jj}(s)}{n_{jj}(s)} = \frac{s \cdot (s - \eta) \cdot z_{jj,R}(s)}{n_{jj}(s)} \quad (22)
\]

would have to hold, where \( z_{jj,R}(s) \) is an arbitrary polynomial. But since the parametric approach assigns the same denominator polynomials to all elements in the \( j \)-th column, \( n_{jj}(s) = n_{ij}(s) \) holds. With (21), this implies \( \deg \left( z_{ij}(s) \right) \leq \deg \left( z_{jj}(s) \right) = 1 \). However, (22) requires \( \deg \left( z_{ij}(s) \right) \geq 2 \) and by this contradiction, \( g_{r_f}(s) = 0 \forall s \in C \) is proven.

Since invariant zeros are seldom non-interconnecting zeros, we virtually augment the system (4). As depicted in Fig. 2, additional internal dynamics are added. The purpose is to transform the cRHP zeros into non-interconnecting zeros by properly selecting \( \bar{A} \in C^{\nu \times \nu}, \bar{C} \in C^{n_y \times \nu} \), and the order \( \nu \) of the additional dynamics. Furthermore, the faults are reordered by a permutation matrix \( F \in \mathbb{R}^{n_f \times n_f} \) such that \( f = \bar{F} \bar{f} \). The resulting augmented system with \( z = [x^T \ \pi]^T \) reads

\[
\dot{\bar{x}} = \begin{bmatrix} A & 0 \\ 0 & \bar{A} \end{bmatrix} \bar{x} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} E_aF \\ 0 \end{bmatrix} \bar{f}, \quad (23a)
\]

\[
\bar{y} = \begin{bmatrix} C \ \\ \bar{C} \end{bmatrix} \bar{x} + E_s \bar{f}. \quad (23b)
\]

Note that the additional internal dynamics might be unstable, since they are only needed for designing a stable FIO for the augmented system (23). They do not have to be implemented.

With the Rosenbrock matrix \( \Sigma(\eta) \) for the augmented system we have

\[
\det (\Sigma(\eta)) = \det (\eta I_n - \bar{A}) \cdot \det (\eta I_n - A) \cdot \det (E_aF + E_s) \quad (24)
\]

which can readily obtained by row- and column permutations in \( \det (\Sigma(\eta)) \). Hence, the eigenvalues of \( \bar{A} \) are invariant zeros of the augmented system.

In the following, consider an invariant zero of (4) with multiplicity 1, evaluate (16a), and let \( \alpha \) be the number of non-zero elements in \( z_f \). Due to the relation used in (24), this number is the same for the system with permuted faults. Therefore, it is always possible to find a matrix \( F \), such that the first \( \alpha \) elements of \( \bar{F} \) differ from 0, i.e.,

\[
z_f = \bar{F} \pi_f = \bar{F} \cdot [\pi_{f,1} \cdots \pi_{f,\alpha} \ 0 \cdots 0]^T. \quad (25)
\]

After calculating \( F \), we choose the order of the additional dynamics to be \( \nu = \alpha - 1 \) and select

\[
\bar{A} = \eta I_{\alpha-1}. \quad (26)
\]
As a result, \( \eta \) is an invariant zero of the augmented system with multiplicity \( \alpha \). It remains to find \( C \) to guarantee that \( \eta \) is rendered a non-interconnecting zero. Evaluating (16a) \( \alpha \) times for the augmented system (23) and using condition (16b) gives

\[
\begin{bmatrix}
\eta I_n - A & 0 & E_a F \\
0 & 0 & 0 \\
-C & -C & E_b F
\end{bmatrix}
\begin{bmatrix}
z_{x,i} \\
z_{x,b,i} \\
\phi_i
\end{bmatrix} = 0, \quad i = 1, \ldots, \alpha,
\]

(27)

where we partitioned \( z_{x,i} = [z_{x,i}^T z_{x,b,i}^T]^T \). Summarizing all \( \alpha \) conditions with \( Z_{x,t} = [z_{x,t}^T \cdots z_{x,t,\alpha}^T] \), \( Z_{x,b} = [z_{x,b,1} \cdots z_{x,b,\alpha}] \) and \( \mathbb{Z}_f = [I_n \ 0]^T \) results in

\[
\begin{align*}
(\eta I_n - A) Z_{x,t} + E_a F \mathbb{Z}_f &= 0, \quad (28a) \\
-C Z_{x,t} - C Z_{x,b} + E_b F \mathbb{Z}_f &= 0. \quad (28b)
\end{align*}
\]

From (28a) we calculate \( Z_{x,t} = -(\eta I_n - A)^{-1} E_a F \mathbb{Z}_f, \) which exists since \( \eta \) is not an eigenvalue of \( A \) by assumption 4. Inserting this into (28b) results in the condition

\[
(C (\eta I_n - A)^{-1} E_a + E_s) F \mathbb{Z}_f = 0
\]

(29)

on \( C \) and \( Z_{x,b} \). Due to basic linear algebra, this equation is solvable if and only if

\[
\text{rank}(C) = \text{rank}(C(\eta)) \quad (30)
\]

holds. Due to (16), \( (C (\eta I_n - A)^{-1} E_a + E_s)z_f = 0 \) holds and with (25) we conclude

\[
(C (\eta I_n - A)^{-1} E_a + E_s) F \mathbb{Z}_f = 0
\]

(31)

which implies \( \text{rank}(\Theta(\eta)) < \alpha \). Thus we select \( C \) as a linear combination of the linearly independent columns of \( \Theta(\eta) \), i.e.

\[
C = \Theta(\eta)A,
\]

(32)

with any full rank matrix \( A \in \mathbb{R}^{\alpha \times (\alpha - 1)} \).

**B. DFIO parameterization**

In case of complex conjugate pairs of cRHP zeros, \( \overline{A} \) and \( \overline{C} \) are complex. For these cases, a similarity transformation \( A' = T AT^{-1}, \ B' = TBT \), \( E_a' = T E_a \), \( C' = CT^{-1}, \) and \( E_s = E_s \) is introduced to enable implementation of the DFIO as shown in the following lemma.

**Lemma 1:** Given a pair of invariant zeros \( \eta_{1,2} = a \pm jb \) with arbitrary \( \alpha \geq 2 \) as introduced in (25) and \( A_1 = A_2 = A \) for the associated system augmentations as in (32). Then the similarity transformation using

\[
T^{-1} = \begin{bmatrix}
I_n & 0 \\
0 & T_{mr}^{-1}
\end{bmatrix}, \quad T_{mr}^{-1} = \begin{bmatrix}
1 & -j \\
1 & j
\end{bmatrix} \otimes I_{\alpha - 1}
\]

(33)

results in real matrices \( A', E_a', C', \) and \( E_s' \).

**Proof:** For \( A' \) we calculate

\[
A' = \begin{bmatrix}
I_n & 0 \\
0 & T_{mr}
\end{bmatrix} \begin{bmatrix}
A & 0 \\
0 & \overline{A}
\end{bmatrix} \begin{bmatrix}
I_n & 0 \\
0 & T_{mr}^{-1}
\end{bmatrix} = \begin{bmatrix}
A & 0 \\
0 & T_{mr} A T_{mr}^{-1}
\end{bmatrix}.
\]

(34)

Due to the design procedure presented in Section IV-A, \( \overline{A} \) is given by

\[
\overline{A} = \begin{bmatrix}
a+jb & 0 \\
0 & a-jb
\end{bmatrix} \otimes I_{\alpha - 1}.
\]

(35)

For invertible matrices of appropriate dimensions, \( (P \otimes I)(Q \otimes I) = (PQ) \otimes I \) and \( (P \otimes Q)^{-1} = P^{-1} \otimes Q^{-1} \) hold and thus

\[
T_{mr} A T_{mr}^{-1} = \begin{bmatrix}
1 & -j \\
1 & j
\end{bmatrix}^{-1} \begin{bmatrix}
a+jb & 0 \\
0 & a-jb
\end{bmatrix} \otimes I_{\alpha - 1} = \begin{bmatrix}
a+b & -ab \\
-ba & a
\end{bmatrix} \otimes I_{\alpha - 1},
\]

(36)

(37)

which implies that \( A' \) is a real matrix.

Since \( E_a' = E_a = E_s F \), the sensor fault matrix for the augmented system is obviously real. The transformed actuator fault matrix reads

\[
E_a' = \begin{bmatrix}
I_n & 0 \\
0 & T_{mr}
\end{bmatrix} \begin{bmatrix}
E_a F & 0
\end{bmatrix} = \overline{E}_a
\]

(38)

and thus is also real. \( B' \) can be shown to be real by similar arguments.

To proof that \( C' \) is real, consider the augmented output matrix after the two system extensions due to \( \eta_1 \) and \( \eta_2 \),

\[
\overline{C} = \begin{bmatrix}
C & \Theta_1(\eta_1)A & \Theta_2(\eta_2)A
\end{bmatrix}.
\]

(39)

Therein, \( \Theta_1(\eta_1) \) is given in (29). After the first system augmentation, \( \Theta_2(\eta_2) \) can be computed as shown on the bottom of this page in (40). Since \( \eta_1 \) and \( \eta_2 \) form a complex conjugate pair, we thus conclude that \( \Theta_2(\eta_2) \) is the complex conjugate of \( \Theta_1(\eta_1) \), i.e. \( \Theta_2(\eta_2) = \Theta_1^*(\eta_1) \). This implies that the third column block in (39) is this complex conjugate of the second column block. Thus in

\[
[\Theta_1(\eta_1)A \quad (\Theta_1(\eta_1)A)^*] \in \mathbb{C}^{n_f \times 2(\alpha - 1)},
\]

(41)

the \( i + \alpha - 1 \)-th column is the complex conjugate of the \( i \)-th column and

\[
[\Theta_1(\eta_1)A \quad (\Theta_1(\eta_1)A)^*] \begin{bmatrix}
1 & -j \\
1 & j
\end{bmatrix} \otimes I_{\alpha - 1}
\]

(42)
is a real matrix. Therefore $C'$ is a real matrix and the proof is completed.

For the resulting augmented and transformed system

$$\dot{z}' = A' z' + B' u + E_a' f,' \quad (43a)$$
$$y' = C' z' + E_c f, \quad (43b)$$
a static FIO $(L', V')$ is designed using the parametric approach described in Section III, where only minimum-phase zeros are rendered uncontrollable using (15) and all other parameter vectors are assigned by (14). To determine the denominator degree of each diagonal element in $G_{rf}(s)$, let $\kappa_i$ be the number of cRHP zeros for which the $i$-th element in $z_f$ is non-zero (cf. (16)). Then each channel $g_{r_i f_i}(s)$ is assigned $\delta_i + \kappa_i$ poles. The resulting FIO for the augmented system is given by

$$\dot{z}' = \left( A' - L' C' \right) \dot{z}' + B' u + L' y', \quad (44a)$$
$$r = V' \left( y' - y \right). \quad (44b)$$

Introducing the change of variables $\xi = x - \bar{x}'$ and partitioning

$$L' = \begin{bmatrix} L_1' \\ L_2' \end{bmatrix} \quad (45)$$
with $L_1' \in \mathbb{R}^{n \times n_z}$, $L_2' \in \mathbb{R}^{r' \times n_y}$, (44) can be implemented as

$$\dot{x} = A \dot{x} + B u + v, \quad (46a)$$
$$v = L_1' C' \xi + L_1' \left( y - C \bar{x} \right), \quad (46b)$$
$$\dot{\xi} = \left( A' - L_2' C' \right) \xi + \left( -L_2' \right) \left( y - C \bar{x} \right), \quad (46c)$$
$$r = V_1' \dot{\xi} + V_2' \left( y - C \bar{x} \right), \quad (46d)$$

which is a DFIO as presented in (3). Therein, the order of the internal observer dynamics is $\nu = \sum_{k=1}^{\mu_{cRHP}} \alpha_k - 1$, where $\alpha_k$ is the number of non-zero elements in $\mathbf{z}_f$ in (25) for the $k$-th system augmentation. To summarize the entire design procedure we recall Fig. 1. The system is repeatedly augmented by (26) and (32) to create non-interconnecting zeros. For each complex conjugate pair of cRHP zeros, a similarity transformation (33) is employed to render the matrices of the augmented system real. The resulting virtual system enables the design of a static FIO (12), (13) using the parametric approach and the resulting FIO can be realized as a DFIO (3) for the original system.

Remark 1: While the system augmentations presented in Section IV are necessary for the design of stable fault isolation observers for non-minimum phase systems, we emphasize that their application is beneficial for minimum phase systems under certain circumstances as well. If a system contains invariant zeros in the open left complex half-plane but close to the imaginary axis, a static FIO obviously suffers from long settling times if $e(t = 0) = x(0) - \bar{x}(0) \neq 0$. A DFIO can achieve faster settling times because the invariant zeros do not have to be compensated by slow eigenvalues. Furthermore, if a system contains invariant zeros in the far left of the complex half-plane, the resulting fast eigenvalues when employing static FIOs might result in poor robustness with respect to high-frequency disturbances such as sensor noise. DFIOs provide better disturbance attenuation in these cases by avoiding the compensation of such invariant zeros.

Remark 2: If a system has singular $D^*$ and is non-minimum phase at the same time, the approaches of Section IV and [25] can be combined. In a first system augmentation, the singularity of $D^*$ is dealt with and in a second step, the system is further augmented to transform all cRHP zeros into non-interconnecting zeros as presented in this article.

V. EXAMPLE

To demonstrate the applicability of the results, we employ the proposed approach in the observer-based fault isolation of a gantry crane. It can be modeled as a fourth-order system with crane position and velocity and pendulum angle and angular velocity as system states. While the first fault is a sensor fault in the measurement of the pendulum angle, the second fault is an actuator fault in the input voltage of the DC-motor driving the crane. With the parameters of our lab experiment, the system matrices read

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -70.31 & -0.97 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -70.31 & -11.75 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 5.89 \\ 0 \\ 5.89 \end{bmatrix}, \quad (47a)$$
$$E_a = \begin{bmatrix} 0 & B \end{bmatrix}, \quad E_s = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad (47b)$$
$$C = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \quad (47c)$$

Due to the actuator and sensor fault, the system $(A, E_a, C, E_s)$ contains a pair of complex conjugate invariant zeros at $\eta_{1,2} = \pm 3.28 j$, i.e. $\mu_{cRHP} = 2$. Furthermore, $(A, C)$ is fully observable, $D^*$ is regular and the Assumptions 1–5 are fulfilled. Following Section IV, the system is augmented such that all invariant zeros become non-interconnecting zeros. Because of $\mu_{cRHP} = 2$, two augmentation loops are needed. For each invariant zero we have $\alpha = 2$ and $F = I_2$. Furthermore, $\Lambda = [1 5]^{-1}$ is chosen. Since the system contains two complex conjugate invariant zeros, the transformation described in Lemma 1 is employed resulting in

$$A' = \begin{bmatrix} 0 & 3.28 \\ -3.28 & 0 \end{bmatrix}, \quad C' = \begin{bmatrix} 0 \\ 62.68 \\ 0 \end{bmatrix}. \quad (48)$$

Assigning poles $\lambda_{1,1} = -3.2$ and $\lambda_{1,2} = -3.3$ to the first transfer channel and $\lambda_{2,1} = -8$, $\lambda_{2,2} = -8.2$, $\lambda_{2,3} = -8.5$, and $\lambda_{2,4} = -9$ to the second channel, the parametric FIO design described in Section III is employed for the virtual augmented system with a static gain of $1$ for each channel.
In the experiment, an abrupt sensor fault occurs at $t = 1s$, i.e. there is a constant offset in the measurement of the pendulum angle. After $t = 2s$, a simultaneous actuator fault is present. During the experiment, a constant input $u$ is active. Fig. 3 visualizes the results of the simulation and the lab experiment. Note that despite the simplified model and sensor noise, the measured residuals are very close to the simulation results and fault isolation is achieved. Both residuals exhibit typical non-minimum phase behavior.

VI. CONCLUSION

In this contribution we provided a design procedure for dynamically extended observers, which allows to isolate faults in linear non-minimum phase systems. The method applies to both actuator and sensor faults and can also be employed to improve robustness and/or transient behaviour in systems with invariant zeros in the left complex half-plane. The applicability was demonstrated in both simulations and experiments.

REFERENCES