RATIONAL SYSTEMS AND MATRIX INEQUALITIES FOR MULTICRITERIA VISUAL SERVOING AND VISUAL-BASED LOCALIZATION

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Joint work with
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UFSC, Florianópolis, 2011/03/24
Advent of efficient and cheap visual sensors $\rightarrow$ use of visual information in the robot control algorithms $\rightarrow$ broader field of “reflex actions”

A “multicriteria” problem
- convergence
- visibility constraints
- 3D constraints during the displacement
- actuators saturations
- etc.

highly nonlinear and often uncertain

example: the two basic “position-based” and “image-based” strategies
Example: “Position-based” visual servoing
(a.k.a. “situation-based”/“3D” visual servo)

\[ e = C(r - r^*) \]

A synthesis method

\[ \dot{e} \approx -\lambda e, \quad \lambda > 0, \quad \text{in closed-loop} \]

\[ \dot{r} = B(r) T_c \]

set \( T_c = -\lambda C(r - r^*) \), with \( C = [B(r^*)]^{-1} \) or \( C = [B(\hat{r})]^{-1} \)

Comments

- involves a localization algorithm, time-consuming and error prone
- “approximate control” over the 3D trajectory
- visibility constraint
Example: “Image-based” visual servoing
(a.k.a. “feature-based”/“2D” visual servo)

\[ e = C(s - s^*); \text{ aim: get } \dot{e} \approx -\lambda e, \lambda > 0, \text{ in closed-loop} \]

\[ \dot{s} = J(s, z)T_c \]

\[ T_c = -\lambda C(s - s^*), \text{ with } C = [J(s^*, z^*)]_+ \text{ or } C = [J(s, \hat{z})]_+ \]

Comments:

\( \oplus \) no 3D reconstruction
\( \oplus \) “approximate control” over the 2D trajectory
\( \ominus \) consequent 3D trajectory
\( \oplus \) risk of “local minima”: \( T_c \to 0 \), yet \( s(+\infty) \neq s^* \)
AIMS AND OUTLINE OF THE TALK

Proposal of a generic framework to the multicriteria analysis and synthesis of visual servos (positioning of a 6DOF camera w.r.t. a static target)
- Unification of 3D and 2D multicriteria visual servoing (MVS) in the rational systems framework
- Basics of an “LMI solution” (LMI = Linear Matrix Inequalities) through the Lyapunov theory
- An approach to multicriteria analysis and synthesis based on global linearization techniques and quadratic stability
- A second solution to multicriteria analysis based on differential algebraic representations and more general Lyapunov functions

Statement of the visual-based localization problem as the dual—in the sense of observation/control duality—of 2D visual servoing
- A solution based on global linearization and set-membership filtering

Conclusions and Prospects
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Conclusions and Prospects
UNIFICATION OF 3D/2D MVS

(kinematic context)

Open-loop state space model

Camera velocity screw \( u = (V_x, V_y, V_z, \Omega_x, \Omega_y, \Omega_z)' \)

Relative sensor-target situation \( x = (t_x, t_y, t_z, \lambda, \mu, \nu)' \)

\[
\begin{pmatrix}
\dot{x} \\
y
\end{pmatrix} = f(x, u)
\]

(3D): \( y = x \)
(2D): \( y = s - s^* \)

Theoretical foundations

- state equation: rigid body kinematics
- output equation (for 2D servos): perspective projection of the target's dedicated spots

\[F_S\]
\[O\]
\[T\]
\[F_T\]
\[S\]
\[T_i\]
\[a_i\]
\[b_i\]
\[c_i\]

Image plane
Optical axis
UNIFICATION OF 3D/2D MVS

(kinematic context)

\[
\begin{align*}
(\dot{x}, y) &= f(x, u) \\
(\dot{x}_c, u) &= k(x_c, y)
\end{align*}
\]

Camera velocity screw \( u = (V_x, V_y, V_z, \Omega_x, \Omega_y, \Omega_z)' \)

Relative sensor-target situation \( x = (t_x, t_y, t_z, \lambda, \mu, \nu)' \)

\[ \begin{align*}
(3D) &: y = x \\
(2D) &: y = s - s^*
\end{align*} \]

Multicriteria visual servoing problem statement

- positioning of the camera w.r.t. the target
- stabilization of \( \tilde{x} = (x', x'_c)' \) to 0
- constraints fulfillment
- boundedness of “additional variables” \( \zeta_j = \zeta_j(\tilde{x}) \)
Rational system
\[
\begin{pmatrix}
\dot{x} \\
y
\end{pmatrix} = \begin{pmatrix}
A(x, \chi) & B(x, \chi) \\
C(x, \chi) & D(x, \chi)
\end{pmatrix} \begin{pmatrix}
x \\
u
\end{pmatrix}
\]

- \((x', \chi')' \in \Xi \times \Xi_\chi\)
- \(A(., .), B(., .), C(., .), D(., .)\) rational matrix functions well-defined on \(\Xi \times \Xi_\chi\)

Through a bijective change of variable, e.g.
\[
x = (t_x, t_y, t_z, L = \tan \frac{\Lambda}{2}, M = \tan \frac{\mu}{2}, N = \tan \frac{\nu}{4})',
\]
the open-loop model becomes rational

If the controller is rational, then the problem is stated in terms of
- [the stability analysis of] / [the asymptotic stabilization to] the equilibrium \(\tilde{x}^* = 0\) of the closed-loop \(\dot{\tilde{x}} = \tilde{A}(\tilde{x}, \chi)\tilde{x}\), with \(\tilde{A}(., .)\) rational well-defined on \(\tilde{\Xi} \times \Xi_\chi\), and \(0 \in \tilde{\Xi}\)
- the boundedness \(\zeta_j \in [\zeta_j, \tilde{\zeta}_j]\) of additional variables \(\zeta_j = Z_j \tilde{x}\) or \(\zeta_j = Z_j(\tilde{x}, \chi)\tilde{x}\), with \(Z_j(., .)\) rational well-defined on \(\tilde{\Xi} \times \Xi_\chi\)
Potentialities

- No local minima: $\tilde{x}^* = 0$ is attractive $\Rightarrow s(+\infty) = s^*$
3D/2D MVS & RATIONAL SYSTEMS

★ Potentialities

- No local minima: \( \tilde{x}^* = 0 \) is attractive \( \Rightarrow s(+\infty) = s^* \)
- \( \zeta_j \in [\zeta_j, \bar{\zeta}_j] \) enables to handle multiple constraints
  - avoidance of actuators saturations: \( \zeta_j = u_j \) (e.g. \( |u_j| < \infty \) for \( u = -\lambda J^+(s - s^*) \))
  - fulfillment of visibility constraints (including for 3D servos): \( \zeta_j = s_j - s_j^* \)
  - guarantee of 3D constraints (including for 2D servos): \( \zeta_j = d_{3D} \)
3D/2D MVS & RATIONAL SYSTEMS

★ Potentialities

- No local minima: \( \tilde{x}^* = 0 \) is attractive \( \Rightarrow s(+\infty) = s^* \)
- \( \zeta_j \in [\zeta_j, \bar{\zeta_j}] \) enables to handle multiple constraints
- Rational controllers encompass most—not to say all—"classical" controllers
  - Static linear state feedback \( u = Kx \), e.g. \( u = -\lambda B^{-1}(0)x \)
  - Static nonlinear state feedback \( u = K(x)x \), e.g. \( u = -\lambda B^{-1}(x)x \)
  - Static linear output feedback \( u = Ky \), e.g. \( u = -\lambda J^+_{s^*, z^*}(s - s^*) \)
  - Static gain-scheduled output feedback \( u = K(x)y \), e.g. \( u = -\lambda J^+_{s, \tilde{z}}(s - s^*) \)
  - Dynamic linear output feedback \( (\dot{x}_c \ u) = \begin{pmatrix} K_c & K_{cy} \\ K_u(x) & K(x) \end{pmatrix} (x_c \ y) \)
  - Dynamic gain-scheduled output feedback \( (\dot{x}_c \ u) = \begin{pmatrix} K_c(x) & K_{cy}(x) \\ K_u(x) & K(x) \end{pmatrix} (x_c \ y) \)
3D/2D MVS & RATIONAL SYSTEMS

★ Potentialities

- No local minima: $\tilde{x}^* = 0$ is attractive $\Rightarrow s(+\infty) = s^*$
- $\zeta_j \in [\underline{\zeta}_j, \bar{\zeta}_j]$ enables to handle multiple constraints
- Rational controllers encompass most—not to say all—“classical” controllers
- Extensions: in addition to handling various uncertainties, contexts where the dynamics cannot be neglected, etc.
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 Conclusions and Prospects
BASICS OF A LYAPUNOV-BASED SOLUTION

Aim: define \( V(., .) = V_P(., .) \) on \( \tilde{\Xi} \times \Xi_\chi \) such that \( \mathcal{E} \triangleq \{ \tilde{x} : V(\tilde{x}, \chi) \leq 1, \forall \chi \in \Xi_\chi \} \) is a “multicriteria basin of attraction”

- local asymptotic stability of the equilibrium state vector \( \tilde{x}^* = 0 \) of \( \dot{\tilde{x}} = \tilde{A}(\tilde{x}, \chi)\tilde{x} \)
  - \( V(\tilde{x}, \chi) > 0 \) on \( \tilde{\Xi} \times \Xi_\chi \setminus \{(0, \chi)\} \); \( V(0, \chi) = 0 \)
  - \( \dot{V}(\tilde{x}(t), \chi) < 0 \) on \( \tilde{\Xi} \times \Xi_\chi \)

- \( \mathcal{E} \) basin of attraction of \( \tilde{x}^* = 0 \) for the unconstrained problem
  - \( \mathcal{E} \subset \tilde{\Xi} \)

- incorporation of the constraints
  - \( \zeta_j \leq Z_j(\tilde{x}, \chi)\tilde{x} \leq \bar{\zeta}_j, \forall (\tilde{x}, \chi) \in \tilde{\Xi} \times \Xi_\chi : V(\tilde{x}, \chi) \leq 1 \)
BASICS OF A LYAPUNOV-BASED SOLUTION

★ Other arguments guiding the search
- constraint $\tilde{x}(0) \in \mathcal{X}_0$ on the initial state vector $\mathcal{X}_0 \subset \mathcal{E}$
- heuristic maximization of the “size” of $\mathcal{E}$: $\min_P f(P)$
- heuristic maximization of the extent of $\mathcal{E}$ towards $\lambda_i \in \tilde{\Xi}$: $\min_P \max_{\chi} V(\lambda_i, \chi)$
- maximization of the closed-loop convergence rate: $\max_{\alpha, P} \alpha$ s.t. $\dot{V}(\tilde{x}(t), \chi) < -2\alpha V(\tilde{x}(t), \chi)$

★ Difficulties raised by Robotics
- the admissible sets $\mathcal{A}_{\zeta_j}$ may not be symmetric w.r.t. 0, nor convex, nor even connected
  - ensuring $V(\tilde{x}, \chi) \leq 1$, $\forall \tilde{x} : \zeta_j \leq Z_j(\tilde{x}, \chi) \tilde{x} \leq \overline{\zeta}_j$ may be too conservative (pessimistic)
- depending on the form of $V_P(., .)$
- independently compute $(\mathcal{E})_r$ for $(\tilde{\Xi} \times \Xi_\chi)_r$, $(\lambda_i)_r$, .... then define $\mathcal{E} \triangleq \cup_r (\mathcal{E})_r$
MATRIX INEQUALITIES

\( \mathcal{L}(X) : A_0 + x_1 A_1 + \cdots + x_n A_n < 0 \) is an LMI (Linear Matrix Inequality) on the decision variable \( X = (x_1, \ldots, x_n)' \), \( A_i = A_i' \) being given matrices ("\( M < 0 \)" \( \equiv \) "\( M \) negative definite")

- Example: \( \mathcal{L}(P, S, G) : \left( \begin{array}{c|cccc} A'P + PA + C_q' S C_q & PB_p + C_q' S D_{qp} + C_q' G \\ \hline & D_{qp}' S D_{qp} + S + D_{qp} G + G' D_{qp} \end{array} \right) < 0 \)

- Important fact: \( \mathcal{L}(X) \) is a convex constraint on \( X \)

\( \rightarrow \) the feasibility of \( \{ \mathcal{L}_1(X), \ldots, \mathcal{L}_N(X) \} \), the minimum of a convex criterion on \( X \) under \( \{ \mathcal{L}_1(X), \ldots, \mathcal{L}_N(X) \} \) can be numerically solved in a polynomial time with an arbitrary precision \( \rightarrow \) such problems are considered as solved

\( \mathcal{B}(X_a, X_b) \triangleq F_0 + \sum_{i=1}^{m_a} x_{ai} F_{ai} + \sum_{j=1}^{m_b} x_{bj} F_{bj} + \sum_{i=1}^{m_a} \sum_{j=1}^{m_b} x_{ai} x_{bj} F_{ij} < 0 \) is a BMI (Bilinear Matrix Inequality) on \( X_a = (x_{a1}, \ldots, x_{am_a})' \) and \( X_b = (x_{b1}, \ldots, x_{bm_b})' \); \( F_0, F_{ai}, F_{bj} \) and \( F_{ij} \) being given

- BMI problems are difficult.

Special case: \( \text{CCP}(P, Q, S, T, X, Y, \ldots) = \{ \mathcal{L}(P, Q, S, T, X, Y, \ldots) \} \cup \{ PQ = I, ST = I \} \)
**An important lemma: the \( S \)-procedure**

- Let \( f_i(\xi) \triangleq (\xi) \, F_i (\xi) \), \( i = 0, \ldots, p \), be given \n
quadratic functions, with \( F_i = F_i' \)

- \( f_0(\xi) \leq 0 \) for all \( \xi \) such that \( f_1(\xi) \leq 0, \ldots, f_p(\xi) \leq 0 \)

\[
\exists \tau_1 \geq 0, \ldots, \tau_p \geq 0 : \forall \xi, \ f_0(\xi) - \sum_{i=1}^{p} \tau_i f_i(\xi) \leq 0
\]

- If the vector \( \xi \) is free in other respects, then this sufficient condition turns to the feasibility
of \( \mathcal{L}(\tau_1, \ldots, \tau_p) : \begin{cases} \tau_1 \geq 0, \ldots, \tau_p \geq 0 \\ F_0 - \sum_{i=1}^{p} \tau_i F_i \leq 0 \end{cases} \)

- the equivalence holds for \( p = 1 \) if \( \exists \xi_0 : f_1(\xi_0) < 0 \)

- Example

Define \( \mathcal{E}_i \triangleq \{ \xi : f_i(\xi) = (\xi)_1 \, F_i (\xi)_1 \leq 0 \} \)

\[
\exists \tau_1 \geq 0, \exists \tau_2 \geq 0 : F_0 - \tau_1 F_1 - \tau_2 F_2 \leq 0 \Rightarrow (\mathcal{E}_1 \cap \mathcal{E}_2) \subset \mathcal{E}_0
\]
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“Global linearization” [Boyd et al.]: “embedding” of $[\text{NLCL}] : \dot{x} = \tilde{A}(\tilde{x}, \chi)\tilde{x}$ into $[\text{LPV}] : \dot{\tilde{x}} = \tilde{A}(\delta)\tilde{x}$, with $\delta = (\delta'_x, \delta'_\chi)' \in \tilde{\Xi} \times \Xi\chi$ a time-varying uncertain parameters vector

$\rightarrow$ the trajectories $\tilde{x}$ of $[\text{NLCL}]$ which wholly lie in $\tilde{\Xi}$ are trajectories of $[\text{LPV}]$

$\rightarrow$ $V(., .)$ establishes the (global) stability of $[\text{LPV}]$

$\Rightarrow$ $V(., .)$ is a Lyapunov function for $[\text{NLCL}]$ on $\tilde{\Xi} \times \Xi\chi$

$\star$ Through existing techniques [El Ghaoui et al.] and minor enhancements

- Rewriting of the $[\text{LPV}]$ into an LFT $F_u(\Sigma, \Delta(t))$
- Search for a Lyapunov function $V(\tilde{x}) = \tilde{x}'P\tilde{x}$, $P > 0$, for $F_u(\Sigma, \Delta(t))$
- Incorporation of the constraints on $E \triangleq \{\tilde{x} : \tilde{x}'P\tilde{x} \leq 1\}$ through the $S$-procedure
  - bassin of attraction: $E \subset \tilde{\Xi}$
  - constraints: $E \subset A_{\zeta_j}$
  - initial conditions: $X_0 \subset E$
A glance at the consequent feasibility/optimization problems

If possible, \( \min(-\alpha) / \min(\text{trace}(P)) / \min(\log(\det(Q^{-1}))) \), under the Matrix Inequalities

<table>
<thead>
<tr>
<th>Convergence</th>
<th>IC</th>
<th>Actuators Saturations</th>
<th>2D and 3D Method #1</th>
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<th>Controller reconstruction</th>
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</thead>
<tbody>
<tr>
<td>( \dot{V} &lt; \alpha V )</td>
<td>( \mathcal{E} \subset \Xi )</td>
<td>( \mathcal{L}(P) )</td>
<td>( \mathcal{L}(P)/\mathcal{L}(P, S_j, G_j) )</td>
<td>( \mathcal{L}(P) )</td>
<td>( \mathcal{L}(P, S_j, G_j) )</td>
</tr>
<tr>
<td>Analysis</td>
<td>( \mathcal{L}_\alpha(P, S, G) )</td>
<td>( \mathcal{L}(P) )</td>
<td>( \mathcal{L}(Q) )</td>
<td>( \mathcal{L}(Q) )</td>
<td>( \mathcal{L}(Q, T_j, H_j) )</td>
</tr>
<tr>
<td>( u = Kx )</td>
<td>( \mathcal{L}_\alpha(Q, Y, T, H) )</td>
<td>( \mathcal{L}(Q) )</td>
<td>( \mathcal{L}(Q, Y) )</td>
<td>( \mathcal{L}(Q) )</td>
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</tr>
<tr>
<td>( u = Kx x )</td>
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<tr>
<td>( u = Ky )</td>
<td>( B(P, K, S, G, \alpha) )</td>
<td>( \mathcal{L}(P) )</td>
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<td>( \mathcal{L}(P, T_j, H_j) )</td>
</tr>
<tr>
<td>( \mathcal{L}_\alpha(P, Q_x, Y, S_x, T_x) ) &amp; ( S_n T_n = I )</td>
<td>( \mathcal{L}(Q_x) )</td>
<td>( \mathcal{L}(P_x, Q_x, Y) )</td>
<td>( \mathcal{L}(Q_x) )</td>
<td>( \mathcal{L}(Q_x, T_j, S_j) )</td>
<td>( \mathcal{L}(K_G) )</td>
</tr>
</tbody>
</table>
CASE STUDY: 3DOF CAMERA

Synthesis of a 3D (static) visual servo of a 3DOF camera: \( u = (V_y, V_z, \Omega_x)' \), \( x = \tilde{x} = (t_y, t_z, L)' \)

- Planar target, fitted with 4 coplanar spots describing a rectangle
- Reference final situation such that the projection of the target is centered and the \( z_T \)-axis is orthogonal to the target plane
- Loose visibility constraint – Reasonable 3D and actuators constraints
- Maximization of the convergence rate \( \alpha \)

\[ \text{unfeasible problem for } |x(0)| \gtrsim |x^0| \text{ with } x^0 = (0.5, 0.8, 0.1)'! \]

\[ \text{solution at } x^0: u = Kx, \text{ with} \]

\[ K = \begin{pmatrix} 1.88 & -0.63 & 2.03 \\ -0.18 & 0.95 & 4.55 \\ -4.97 & -1.98 & 41.99 \end{pmatrix} \quad \alpha = 0.3810 \]
CASE STUDY: 3DOF CAMERA

Geometrization

Actuators constraints
3D constraints
Convergence
Image constraints

Image constraints

RATIONAL SYSTEMS AND MATRIX INEQUALITIES FOR MULTICRITERIA VISUAL SERVOING AND VISUAL-BASED LOCALIZATION

P. Danès
Analysis of a 2D (static) visual servo of a 2DOF camera: \( u = (V_z, \Omega_z)' \) and \( x = \tilde{x} = (t_z, N)' \)

- 2-spot target – At the reference final situation, \( z_T \perp \) target axis
- Constraints: visibility and actuators
- Control law \( u = -\lambda [J(s^*, z^*)]^+(s - s^*), \lambda = 0.1, \) with \( J(., .) \) such that \( \dot{s} = J(s, z)u \)
- Maximization of \( E' \)’s “size”
TEMPORARY CONCLUSION

★ Very pessimistic results!

★ Output feedback (i.e. 2D servos) synthesis unfeasible w/o returning to BMIs...

★ Sources of conservatism
  • Global linearisation
    ▸ increases along with the extent of $\tilde{\Xi}$
    ▸ $\tilde{\Xi}$ is necessarily a parallelotope
    ▸ /!\ well-posedness problems!
  • Quadratic stability
    ▸ a single Lyapunov function is used to conclude on all the realizations of [LPV]
    ▸ moreover, an outer approximation of [LPV] is considered
      · need to search for a minimal LFT $\mathcal{F}_u(\Sigma, \Delta(t))$ of [LPV]
      · etc.
  • Asymmetry and nonconvexity of the admissible sets w.r.t. the constraints

→ how can this last problem, raised by the robotics context, be circumvented?
Augmenting a given multicriteria basin of attraction $\mathcal{E}$, e.g. $\mathcal{E} \triangleq \bigcup_r (\mathcal{E})_r$, by a subset of a polytope $\Pi$ (hyp: $(\partial \mathcal{E} \cap \Pi)$ is connected)

- Obtention of an asymmetric nonconvex multicriteria basin of attraction, better suited to an inclusion into the admissible subset of the state space
- Reduction of the conservatism due to global linearization – Leads to LMIs
- Cannot be trivially extended to synthesis
- Fairly difficult to implement – Can be hardly applied if $n_{\tilde{x}} > 3$
Application to the 2DOF camera case study: analysis of $u = -\lambda [J(s^*, z^*)]^+(s - s^*)$
Augmenting the state vector by an entry with autonomous stable dynamics

Can lead to an asymmetric multicriteria basin of attraction... 

...which is nevertheless convex

Applies to analysis and synthesis, even for high \( n_\tilde{x} \)

Leads to feasibility/optimization problems of dissimilar complexity (LMIs or CCP) depending on the degrees of freedom introduced
EXTENSION II (ANALYSIS AND SYNTHESIS)

★ Application to the 3DOF camera case study: synthesis of \( u = Kx \)

- Same as previously, but much tighter visibility constraint

★ Note: the synthesis of \( u = K(x)x \) would give even better results
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 Ongoing work and Prospects
A DARs-AND-BQLFs SOLUTION

- Differential-algebraic representations (DARs) and Biquadratic Lyapunov functions (BQLFs) [Trofino] [Coutinho et al.]
  - [NLCL]: \( \dot{\tilde{x}} = \tilde{A}(\tilde{x}, \chi)\tilde{x} \), with \( \tilde{A}(\tilde{x}, \chi) \) rational well-defined \( \iff \exists \) constant \( A_1, A_2 \) and affine \( \Delta_1(\ldots), \Delta_2(\ldots) \) s.t. [DAR]: \[
\begin{aligned}
\dot{\tilde{x}} &= A_1\tilde{x} + A_2\pi \\
0 &= \Delta_1(\tilde{x}, \chi)\tilde{x} + \Delta_2(\tilde{x}, \chi)\pi
\end{aligned}
\]
  - BQLF
    \[
    V(\tilde{x}, \chi) = \tilde{x}'\mathcal{P}(\tilde{x}, \chi)\tilde{x}, \quad \text{with} \quad \mathcal{P}(\tilde{x}, \chi) = \begin{bmatrix}
    \Theta(\tilde{x}, \chi) \\
    I_{\tilde{n}}
  \end{bmatrix}' \begin{bmatrix}
    \Theta(\tilde{x}, \chi) \\
    I_{\tilde{n}}
  \end{bmatrix}
    
    P = P' \text{ to be determined, and } \Theta(\ldots) \text{ affine selected beforehand}

- In the same vein, the constraints write as
  - [C\_j]: \( \zeta_j = Z_j(\tilde{x}, \chi)\tilde{x} \), with \( Z_j(\ldots) \) rational well-defined \( \iff \exists \) constant \( K_{1j}, K_{2j} \) and affine \( \Upsilon_{1j}(\ldots), \Upsilon_{2j}(\ldots) \) s.t. [C\_j]: \[
\begin{aligned}
\zeta_j &= K_{1j}\tilde{x} + K_{2j}\tilde{\pi}_j \\
0 &= \Upsilon_{1j}(\tilde{x}, \chi)\tilde{x} + \Upsilon_{2j}(\tilde{x}, \chi)\tilde{\pi}_j
\end{aligned}
\]
  \( \tilde{\pi}_j = \tilde{\pi}_j(\tilde{x}, \chi); \ Upsilon_2(\ldots) \text{ column full rank on } \tilde{\Xi} \times \Xi_\chi \)
A DARs-AND-BQLFs SOLUTION

Key steps of the solution (apart from maximizing $\mathcal{E}$’s size/extent)

- positive definiteness of $V(.,.)$ on $\tilde{\Xi} \times \Xi_x$
- negativeness of $\dot{V}(.,.)$ on $\tilde{\Xi} \times \Xi_x$
- basin of attraction: $\mathcal{E} \subset \tilde{\Xi}$
- constraints: $\mathcal{E} \subset A_{\zeta_j}$
- initial conditions: $\mathcal{X}_0 \subset \mathcal{E}$

Comments

- $S$-procedure and Finsler’s lemma $\rightarrow$ LMIs
- Leads by nature to an asymmetric nonconvex multicriteria basin of attraction
- Can straightly be extended to more complex Lyapunov functions (polyquadratic, rational, . . .)
- Very large and ill-conditioned LMI problems
- What can be obtained on an elementary problem?

.../...
Analysis of a 2D (static) visual servo of a 2DOF camera: \( u = (V_z, \Omega_z)' \) and \( x = \tilde{x} = (t_z, N)' \)

- 2-spot target – At the reference final situation, \( z_T \perp \) target axis
- Constraints: visibility and actuators
- Control law \( u = -\lambda [J(s^*, z^*)]^+(s - s^*), \lambda = 0.1, \) with \( J(., .) \) such that \( \dot{s} = J(s, z)u \)
- Multicriteria basin of attraction \( \mathcal{E} \triangleq \bigcup_{\{r\}} \mathcal{E}_r \)
  - \( V(x) = (t_z^2, t_zN, N^2, t_z, N)'P(t_z^2, t_zN, N^2, t_z, N) \)
  - \( (\mathcal{E})_r \triangleq \{\tilde{x}: V(\tilde{x}) \leq 1\} \), for selected \( (\Xi)_r \) and \( (\lambda_i)_r \)
CASE STUDY: 2DOF CAMERA & UNCERTAINTY

★ Analysis of a 2D (static) visual servo of a 2DOF camera: \( u = (V_z, \Omega_z)' \) and \( x = \tilde{x} = (t_z, N)' \)

- 2-spot target – At the reference final situation, \( z_T \perp \) target axis
- Constraints: visibility and actuators
- Control law \( u = -\lambda J(s^*, \hat{z}^*)^+(s - s^*), \ldots \) but \( \hat{z}^* = z^*(1 + \chi), |\chi| \leq 0.5, \dot{\chi} = 0 \)
- Multicriteria basin of attraction \( E \triangleq \bigcup \{r\} E_r \)
  \( V(x, \chi) = (t_z^2, t_z N, N^2, t_z, N, \chi)' P(t_z^2, t_z N, N^2, t_z, N, \chi) \)
  \( (E)_r \triangleq \{ \tilde{x} : V(\tilde{x}, \chi) \leq 1, \forall \chi \in \Xi_{\chi} \}, \) for \( (\Xi)_r \) and \( (\lambda_i)_r \)
A LESS PESSIMISTIC ALTERNATIVE THOUGH AT A MODERATED COST?

★ Iso-$V$ of a BQLF, simultaneously “optimized” towards several directions

★ What if a piecewise biquadratic Lyapunov function (PW-BQLF) is used?
AIMS AND OUTLINE OF THE TALK

Proposal of a generic framework to the multicriteria analysis and synthesis of visual servos (positioning of a 6DOF camera w.r.t. a static target)

- Unification of 3D and 2D multicriteria visual servoing (MVS) in the rational systems framework
- Basics of an “LMI solution” (LMI = Linear Matrix Inequalities) through the Lyapunov theory
- An approach to multicriteria analysis and synthesis based on global linearization techniques and quadratic stability
- A second solution to multicriteria analysis based on differential algebraic representations and more general Lyapunov functions

Statement of the visual-based localization problem as the dual—in the sense of observation/control duality—of 2D visual servoing

- A solution based on global linearization and set-membership filtering

Ongoing work and Prospects
Control vs Reconstruct the relative sensor-target situation $x$ from the knowledge of the velocity screw $u$ and of the measurements $y = s - s^*$

Problem statement: recursive prediction of $x$

- given, at time $k$
  - a confidence ellipsoid $\mathcal{E}_k$ enclosing $x(k)$
  - the measurement $y(k) = s(k) - s^*$

Determine a "minimal" $\mathcal{E}_{k+1}$ which encloses $x(k+1)$
VISUAL SERVOING / LOCALIZATION DUALITY

★ Mathematical treatment

• write a rational discrete-time model \([OLz]\) of the open-loop system (e.g. under the ZOH assumption), with a set-membership description of the dynamic and measurement noises
• at each time \(k\):
  ▶ perform the global linearization of \([OLz]\) under the assumption \(x(k) \in \Xi_k\), with \(\mathcal{E}_k \subset \Xi_k\)
  \(\rightarrow [LPV_k]\)
  ▶ (conceptually) isolate the values of \(x(k)\) which are consistent with \(y(k)\) for some realizations of \([LPV_k]\); propagate them until time \(k + 1\) for these realizations
  ▶ get \(\mathcal{E}_{k+1}\) via an LMI program

★ Important aspects

• handling a parametric uncertainty on the model of the target, the intrinsic parameters of the camera, etc.
• to limit the conservatism, search for “minimal” LFTs, which may lead to favor some specific layouts of the visual features
CASE STUDY: 2DOF CAMERA

★ Localization of a motionless camera

- 2-spot or 3-spot target
- With or Without measurement noise

→ cf. forthcoming slides
Influence of the target on the filter's behavior, w/o measurement noise
Influence of the target on the filter's behavior, with measurement noise.
CASE STUDY: 3DOF CAMERA

 Localization of a camera translating onto an horizontal plane and rotating around a vertical axis
AIMS AND OUTLINE OF THE TALK

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  • A solution based on global linearization and set-membership filtering

 Ongoing work and Prospects
CONCLUSION

★ Statement of multicriteria visual servoing (kinematic context) in the rational systems framework
  • “position-based” servo (state feedback) or “image-based” servo (output feedback)
  • constraints: 2D/3D/actuators/…

★ Solution based on the global linearization of the closed-loop system and quadratic stability (analysis and synthesis)
  • conservative results
  • circumvention of some constraints (asymmetry and nonconvexity of the admissible subset of the state space) raised by the robotics problem
  • …but the synthesis of 2D servos requires the solution of BMIs

★ DARs & BQLFs/PQLFs/PW-BQLFs approach (analysis)
  • seems to be more versatile and less conservative

★ Outline of the duality between visual servoing and visual-based localization
ONGOING WORK AND PROSPECTS

⭐ STIC-AMSUD project with Daniel/Luis/Miguel
  • visual based localization via DARs and BQLFs
  • synthesis of visual servos via DARs and BQLFs
  • other types of Lyapunov functions; more complex case studies; comparisons; ...

⭐ Mean term objectives
  • deeper insight into the DAR approach (generalization of the underlying lemma)
  • towards SOS optimization, moments, etc.
  • allowance, rather than avoidance, of saturations (useful in the kinematic context)
  • “bridge the gap” with other developments in visual servoing
    ▷ select (visual features, control schemes) pairs which lead to structural properties, better conditioning, etc.
    ▷ limitation of prior knowledge (target model, etc.)
THANK YOU – GRACIAS – MERCI – OBRIGADO

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