Reduced Order Process Modelling in Self-tuning Control*

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Key Words—Model based self-tuning control; reduced order model; deadtime; frequency response.

Abstract—One major objection to the model-based auto-tuning and self-tuning control is based on the observation by Bristol that a reduced order model being identified would not yield good control of the actual high order process. In this paper we have re-examined the work of Bristol and argued that this observations could not be generalized. We then consider reduced order modelling which incorporates a deadtime element. It is shown that the deadtime could help to provide a close match of the frequency response near the critical frequencies. Extensive simulation studies have also confirmed that reduced order modelling with deadtime included could be an adequate model for practical self-tuning control.

1. Introduction

An essential part of the self-tuning control involves parameter estimation of an explicit or implicit process model (Åström, et al., 1977; Åström and Wittenmark, 1984). In order to reduce the number of parameters to be estimated for practical on-line self-tuning control, the order of the process model chosen is often smaller than that of the actual process hence introducing a potential mismodelling problem. This was highlighted by Bristol in conjunction with his work on a pattern recognition based adaptive control concept (Bristol, 1970, 1977, 1988). The study of mismodelling addresses principally the process structure errors, which are different from parametric errors. Bristol conjectured and demonstrated successfully through some examples that a reduced order model so identified would not yield good process control. Upon this premise, he objected to the concept of self-tuning control based on process model and instead developed the now well-known and commercially successful pattern recognition-based exact adaptive controller.

In this paper we have re-examined the problem of mismodelling. We shall put forward the argument that though Bristol's work and conclusions on mismodelling were relevant and correct, they could not be generalized, particularly when a deadtime element is included. It is inspired and supported by a study of frequency response matching as proposed by Wahlberg and Ljung (1986). The paper is organised as follows. Section 2 summarizes Bristol's work and questions the generality of the results and conclusions drawn. Section 3 presents the study of frequency response matching and simulation results when a deadtime element is included in reduced order modelling. Section 4 discusses the relevance of this study to adaptive control and gives the conclusions drawn.

2. The mismodelling experiments

The details of the mismodelling experiments conducted by Bristol were given elsewhere (Bristol, 1970, 1977). The experiments compared the closed-loop step responses between a high order process controlled by a PI controller and a reduced, second order model identically controlled. The model was derived in two ways: open-loop and closed-loop step response matching.

In essence, Bristol observed that the model derived under open-loop identification gave unacceptable control performance when the loop was closed around the process. Likewise the model derived under closed-loop identification gave grossly different open-loop responses when compared with the original process. However, a convergence to satisfactory control performance could be obtained in the case of closed-loop identification after several iterations. But for the case of the self-tuning regulators, such a convergence to acceptable control performance could not be assured since the self-tuning regulator structure uses an equivalent of open-loop identification structure (Bristol, 1977, 1988).

It is noted that Bristol's experiments utilized step responses as the basis for modelling—in both open-loop and closed-loop identification. Such an approach may not be appropriate as it may not sufficiently excite the relevant natural modes of the process for the purpose of system identification (Åström, et al., 1977). Furthermore, the criterion used in the identification process was the integral square error of the step response; this means that the identification was essentially skewed towards the steady state portion of the set of data points. Therefore, the matching process involved low frequency region of the process's frequency response, again increasing the possibility of mismodelling at the critical frequencies (Wahlberg and Ljung, 1986). Finally, since the matching process involved only the step responses, the results emphasized a magnitude match. There was little consideration for the phase of the process and no attempt to match the model and process phase frequency responses.

From the above discussion, it is not certain that Bristol's observations could be generalized to completely reject the concept of self-tuning control with reduced-order modelling. It has been shown recently by Wahlberg and Ljung (1986) that frequency response matching near the critical point is more appropriate for assessing the accuracy of the reduced order modelling for the purpose of stable feedback control. In the case of least squares parameter estimation, this can be achieved by suitable choice of data filtering to focus on the frequency range of importance. In the following section, we shall attempt another approach involving the use of a deadtime element to address this mismodelling problem.

3. Reduced order modelling with deadtime

In order to facilitate easy comparison, the experiments were conducted on the same process as used by Bristol (Bristol, 1970, 1977):

$$G(s) = \frac{1}{(1 + 0.62s)(1 + 17.2s)}$$

(1)
Fig. 2. Open-loop and closed-loop step responses of process and model B (without deadlines).
Inspired by Wahlberg and Ljung (1986), we shall approach the study of reduced order modelling from a frequency domain perspective. First, we shall consider continuous models identified by matching the time responses, similar to the approach used by Bristol except that a pseudo-random binary sequence (PRBS) test signal is used instead of a step input. Two models were identified. The model, $A$, assumed to be second-order with deadtime, was identified to be:

$$G_m(s) = \frac{e^{-2.43s}}{(1 + 1.98s)(1 + 17.11s)}$$

For comparison, a second model, $B$, assumed to be second-order but without deadtime as in the Bristol's experiment, was identified to have time constants of 1.8 and 16.7. The criterion used for evaluating the goodness of fit for both models was the integral square error between the process and model open-loop responses.

Consider Fig. 1(a) which gives the open-loop step responses of the process and model $A$, and Fig. 2(a) the same for the process and model $B$. Note the closeness of match of the two models to the process. From the process open-loop response alone, one might have accepted the plausibility of representing the process by a second order model without deadtime, if the process was not known to have a higher order structure. However, when the loop was closed around the process and the models, each identically controlled by a proportional-integral (PI) controller, deviations became apparent. These deviations are obvious from Figs 1 and 2 which give the closed-loop responses for three sets of controller settings (proportional gain $K$ and integral time $T_i$) as tabulated in Table 1. These settings were chosen to operate the closed-loop process near optimal, with poor damping and near unstable modes respectively in order to examine if these characteristics could be reflected in the models, as suggested by Bristol (1970, 1977).

The discrepancy between the open-loop and closed-loop responses for model $B$ is evident from the plots, whereas model $A$ can be seen to represent the process sufficiently. To understand the underlying disparity, we examine the Nyquist plots of the process and models as given in Fig. 3. This is

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**Table 1. PI Controller settings**

<table>
<thead>
<tr>
<th>Controller settings</th>
<th>$K$</th>
<th>$T_i$</th>
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</thead>
<tbody>
<tr>
<td>Figs 1(b), 2(b)</td>
<td>2.8</td>
<td>17</td>
</tr>
<tr>
<td>Figs 1(c), 2(c)</td>
<td>5.6</td>
<td>17</td>
</tr>
<tr>
<td>Figs 1(d), 2(d)</td>
<td>6.63</td>
<td>10</td>
</tr>
</tbody>
</table>

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**Fig. 3. Frequency response of process, models $A$ and $B$.**
FIG. 4. Results of discrete time model C.
where the crux of the mismodelling argument resides. In model B, the open-loop response matching did not yield a good match of the frequency response at higher frequencies. But the inclusion of a suitable deadtime element in model A provided a chance not only to match the magnitude but also the phase frequency response of the model to the process resulting in a close match in the Nyquist plot between the process and model A.

This observation confirmed points stated earlier. Firstly, open-loop step responses are poor guides for modelling. They may be used as supplementary guides in the matching process but cannot be the sole basis to arrive at a reduced order model. If anything at all, the frequency response of the process should form the basis for model identification or verification. Secondly, an open-loop step response favours a low frequency identification as evident from the Nyquist plot of model B. However, it is the higher frequency end of the process frequency response, especially the region around its Nyquist frequency, that is of significance in closed-loop control (Wahlberg and Ljung, 1986). This is because closing the loop around the process will create its resonant closed-loop modes that must be modelled. Bristol's observation on mismodelling was due largely to these unmodelled modes.

But for model A, the phase lag introduced by the deadtime overcame this problem of mismodelling. It could be seen from Figs 1(c) and (d) that even in the limiting cases, the model's responses corresponded very closely with those of the process. This points in the direction that a reduced order model with deadtime can be an adequate representation of the original process and hence a basis for control analysis and design.

As practical self-tuning control is invariably implemented in digital form, a discrete time model, C, was also identified using a PRBS excitation signal. The closed-loop responses of the model and process, given in Fig. 4 had the same controller settings as the continuous time case, by means of direct discretization. The simulations confirmed the applicability of the model in the discrete time domain. This point is evident both from the frequency response matching and the closed-loop responses of the model and the process.

Note from the closed-loop response that model C was nonminimum phase. This came about because the deadtime approximated was lower than what the process actually exhibited. From extensive simulation experience, it has been found that as long as the majority of the deadtime has been explicitly modelled, the residual deadtime can be safely reflected as nonminimum phase zero (De Souza et al., 1988) which is automatically identified by the estimator. In discrete identification, variable deadtime can further be identified by overparameterizing the numerator polynomial of the process model.

4. Conclusions

In general, no practical process can be precisely modelled implying, then, all modelling effort is inherently faulty. Therefore, techniques considered should reduce some mismodelling to a minimum. Even with exact knowledge of the process order, a full-order parameter estimator may not be practical due to consideration of the speed of parametric convergence, computational efficiency and precision. On the other hand, a reduced-order model requires only a fixed and small number of parametric estimates. This simpler model not only improves the convergence of the estimates but may also avoid the explosion of sensitive internal process identification data. In the example cited, the reduced-order model has only 3 parameters compared to 8 of the original process. It has been illustrated that the inclusion of a deadtime in the model may be sufficient to compensate for the gross misrepresentation of the process order. This augments the technique of data filtering (Wahlberg and Ljung, 1986) to improve the estimation accuracy near the critical frequencies for closed-loop stability. Thus, the problem of mismodelling which Bristol highlighted becomes much less significant.

The results have direct relevance and application in the area of auto-tuning and self-tuning control. This is because the modelling structure that was employed in the simulation studies is equivalent to that used for auto-tuning and self-tuning, as both essentially make use of open-loop identification.

The main conclusions can be summarized as follows:

(a) Bristol's results based on step-response matching could not be generalized. Frequency response matching near the critical frequencies should instead be used to judge the adequacy of reduced order modelling.

(b) Reduced order modelling with deadtime can compensate for the phase lag disparity between the process and model. This suitably augments the technique of data filtering to ensure that the estimated model could be an adequate basis for the purpose of controller design.

References


